

Dynamics of wealth distribution

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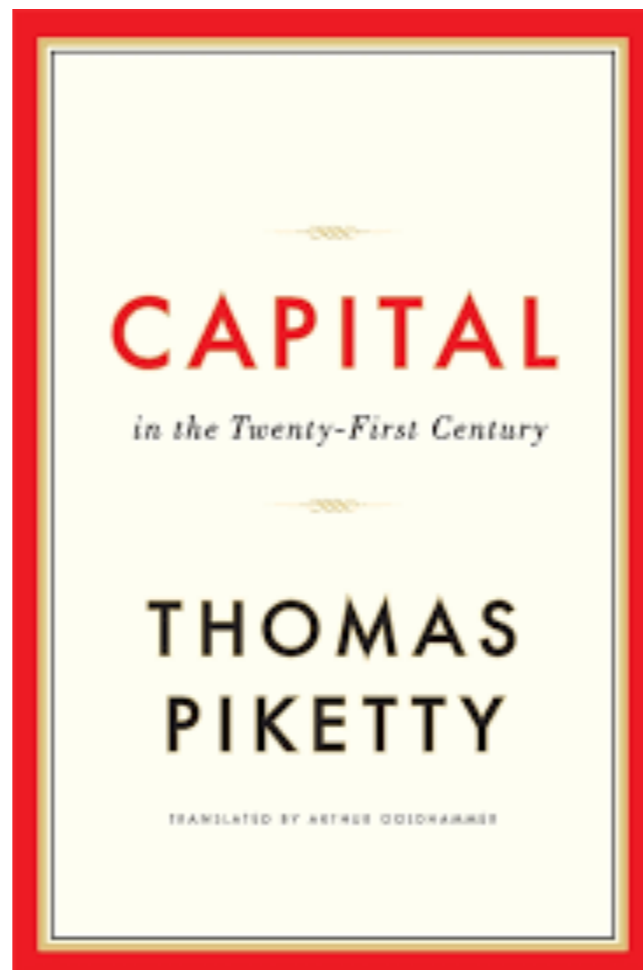
Prologue

Z. Burda, D. Johnston, J. Jurkiewicz, M. Kamiński, M. A. Nowak, G. Papp, and I. Zahed,
Wealth condensation in Pareto macroeconomies, Phys. Rev. E 65, 026102 (2002)

Philip Ball, *Wealth spawns corruption*, Nature News, 28 Jan 2002

<https://www.nature.com/news/2002/020121/full/news020121-14.html>

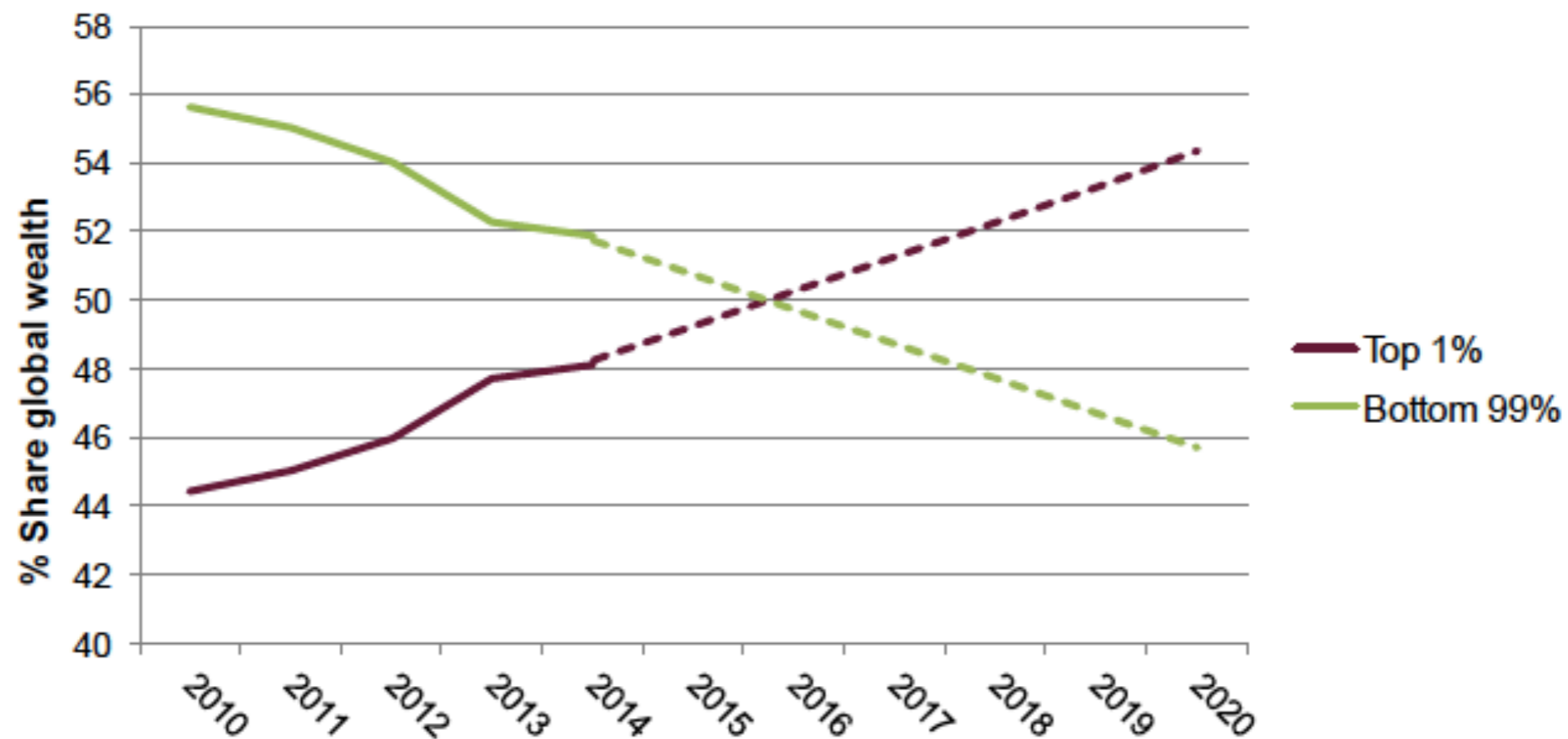
"... the distribution of wealth *is too important an issue to be left to economists, sociologists, historians, and philosophers.* It is of interest to everyone, and that is a good thing. The concrete, physical reality of inequality is visible to the naked eye ..."



Thomas Piketty

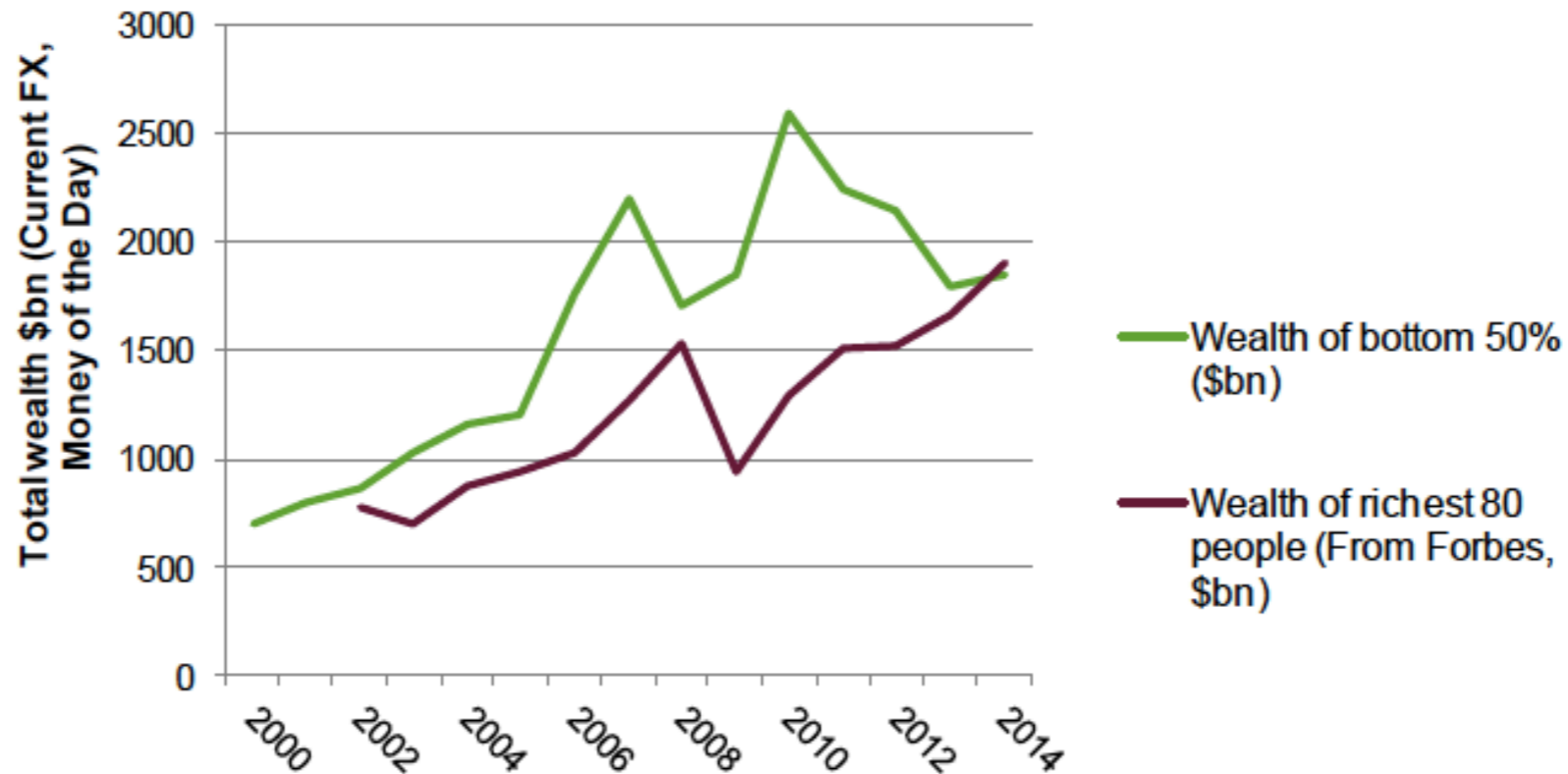
Oxfam issue briefing, Jan 2015 (image source: www.oxfam.org)

Wealth: having it all and wanting more



January 2016 (top 1% = bottom 99%)

Oxfam issue briefing, Jan 2015 (image source: www.oxfam.org)



The number of richest who accumulated the same wealth as the poorest half of the world's population (3.5 billion people)

2010 2011 2012 2013 2014 2015 2016

388 177 159 92 80 62 8

Inequality $r > g$

r = return rate on the capital

g = economy growth rate

" ... When the rate of return on capital significantly exceeds the growth rate of the economy, then it logically follows that inherited wealth grows faster than output and income. People with inherited wealth need save only a portion of their income from capital to see that capital grow more quickly than the economy as a whole. ... "

Piketty's conclusions

- Rich gets richer
- Income tax is not able to reduce the inequality
- Remedy: tax on wealth (tax on capital)

From popular-science to science

T. Piketty, About Capital in the Twenty-First Century, American Economic Review: Papers & Proceedings 105(5): 4853(2015); <http://dx.doi.org/10.1257/aer.p20151060>

T. Piketty and E. Saez, A theory of optimal capital taxation, NBER Working Paper Series, Working Paper 17989, <http://www.nber.org/papers/w17989> (2012);

...

Agent based-modelling

J. Angle, The surplus theory of social stratification and the size distribution of personal wealth, Social Forces 65, 293-326,(1986);

J.-P. Bouchaud and M. Mézard, Wealth condensation in a simple model of economy, Physica A 282, 536 (2000);

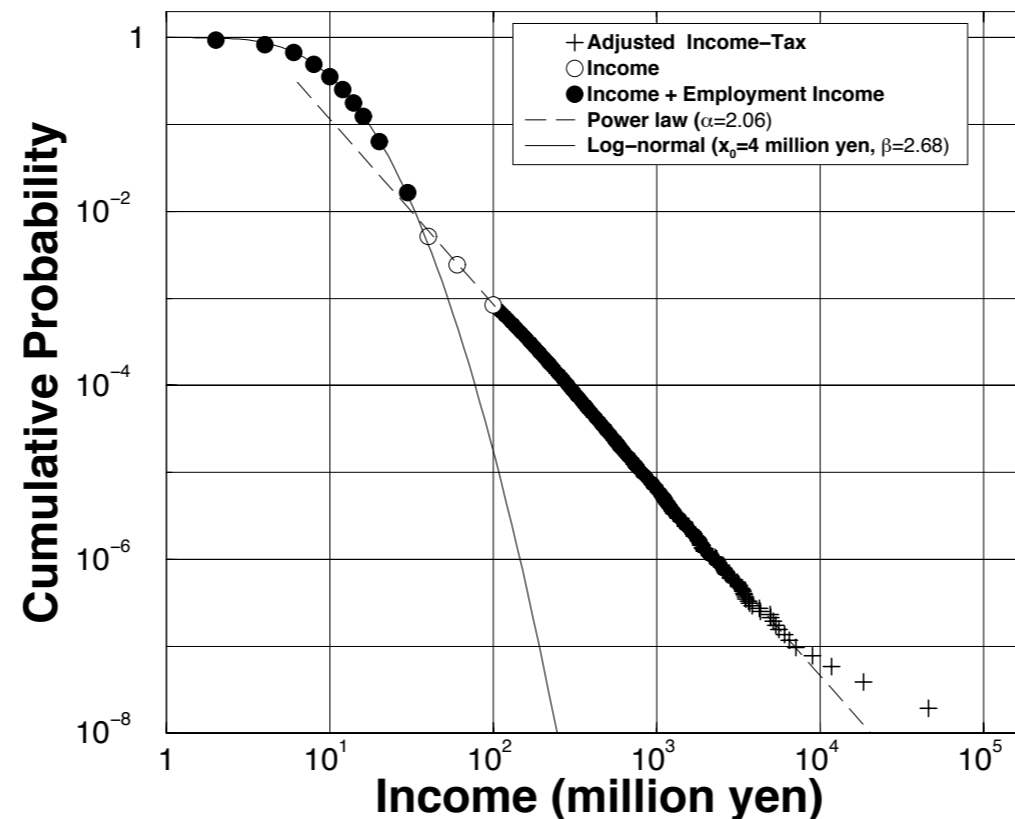
...

Empirical laws on wealth/income distribution

• Gibrat law
$$p(w) \sim \frac{1}{\sqrt{2\pi\sigma^2}w} e^{-\frac{(\ln w - m)^2}{2\sigma^2}} \quad \text{for } w \sim w_*$$

• Pareto law
$$p(w) \sim w^{-1-\alpha} \quad \text{for } w \gg w_*$$

Illustration (Income in Japan 1998, W. Souma, 2002)





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From statistical physics to social sciences / *De la physique statistique aux sciences sociales*

Dynamics of wealth inequality

Dynamique de l'inégalité de richesse

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State of art (milestones)

- Proportionate growth

$$W_{\tau+1} = \lambda_{\tau} W_{\tau}$$

Gibrat law

- Kesten stochastic processes $\langle \lambda \rangle < 1$

$$W_{\tau+1} = \begin{cases} \lambda_{\tau} W_{\tau}, & \text{iff } \lambda_{\tau} W_{\tau} > W_{min} \\ W_{\tau} & \text{otherwise} \end{cases}$$

Pareto law

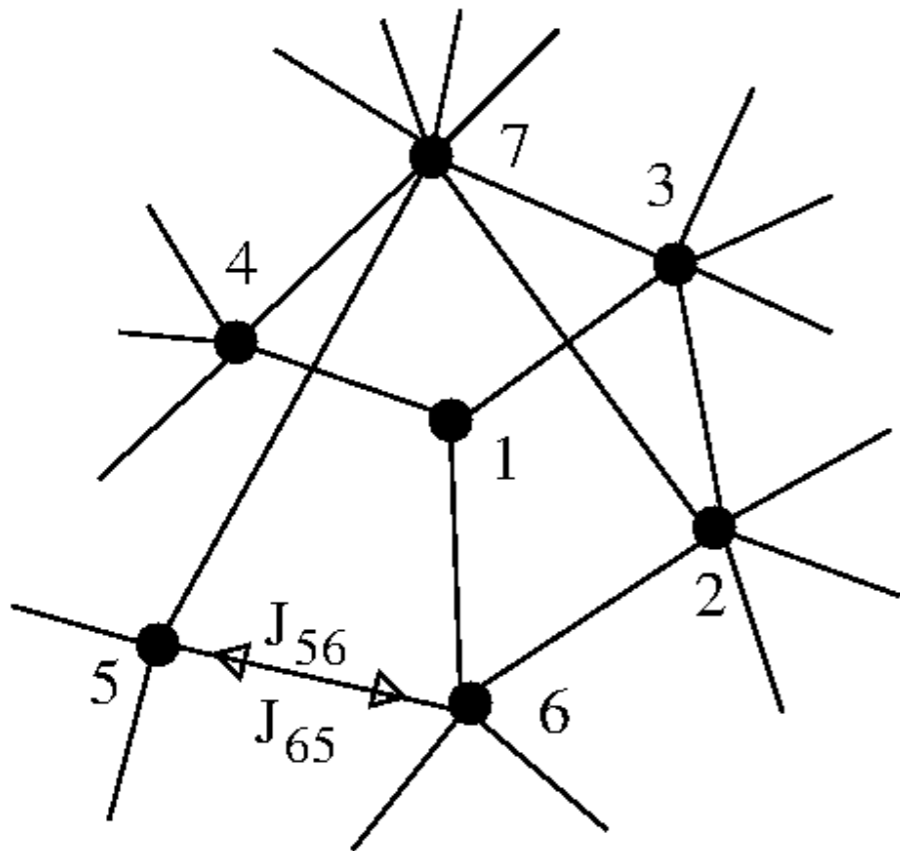
- Agent models

$$W_{i\tau+1} = \lambda_{i\tau} W_{i\tau} + \sum_j F_{ij,\tau}$$

$$F_{ij,\tau} = -F_{ji,\tau}$$

- Taxes + redistribution

Bouchaud-Mezard model



$$W'_{i\tau} = \lambda_{i\tau} W_{i\tau}$$

$$W''_{i\tau} = W'_{i\tau} + \sum_j (J_{ij} W'_{j\tau} - J_{ji} W'_{i\tau})$$

$$W_{i\tau+1} = (1 - \beta) W''_{i\tau} + \frac{\beta}{N} \sum_j W''_{j\tau}$$

- proportionate growth
- exchange (money flow)
- state regulation (tax + redistribution)

Stable system

$$J_{ij} = J_{ji} > 0$$

$$\Delta W_{i\tau} = \sum_j (J_{ij} W_{j\tau} - J_{ji} W_{i\tau}) = \sum_j J_{ij} (W_{j\tau} - W_{i\tau})$$

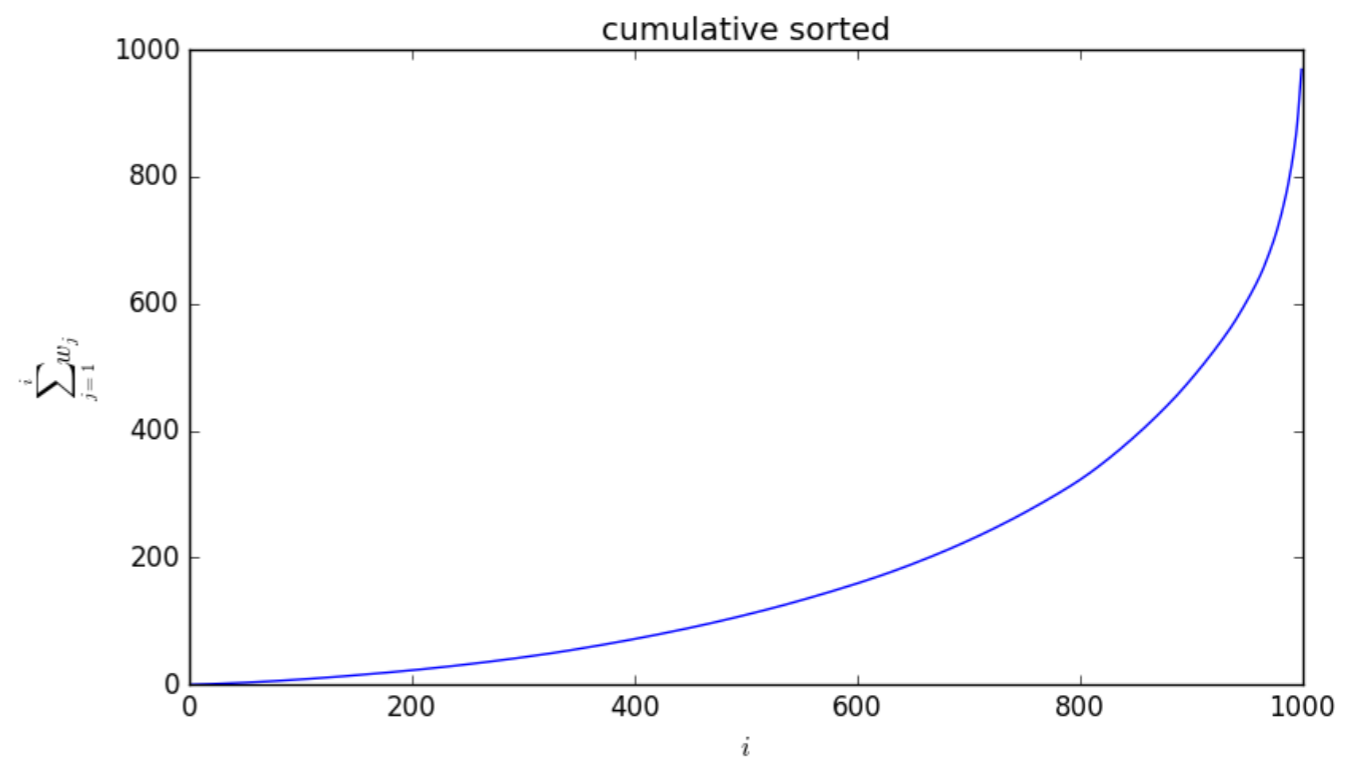
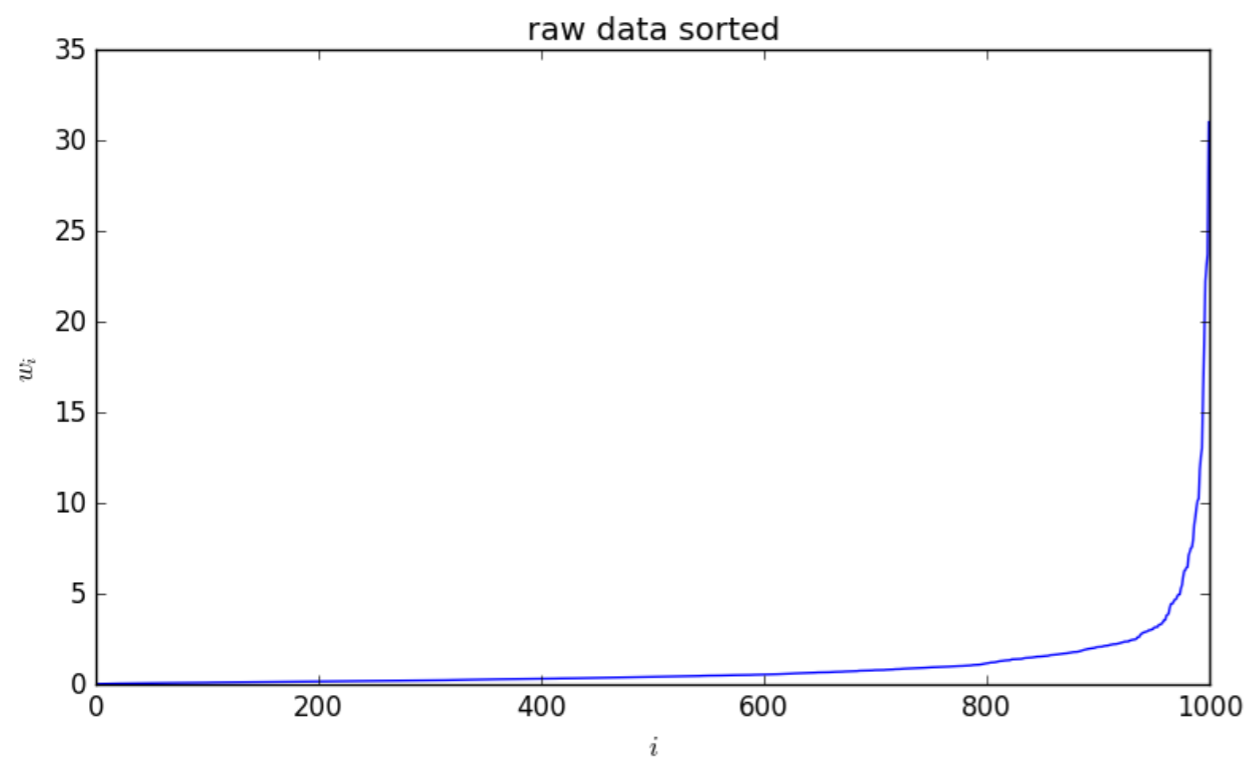
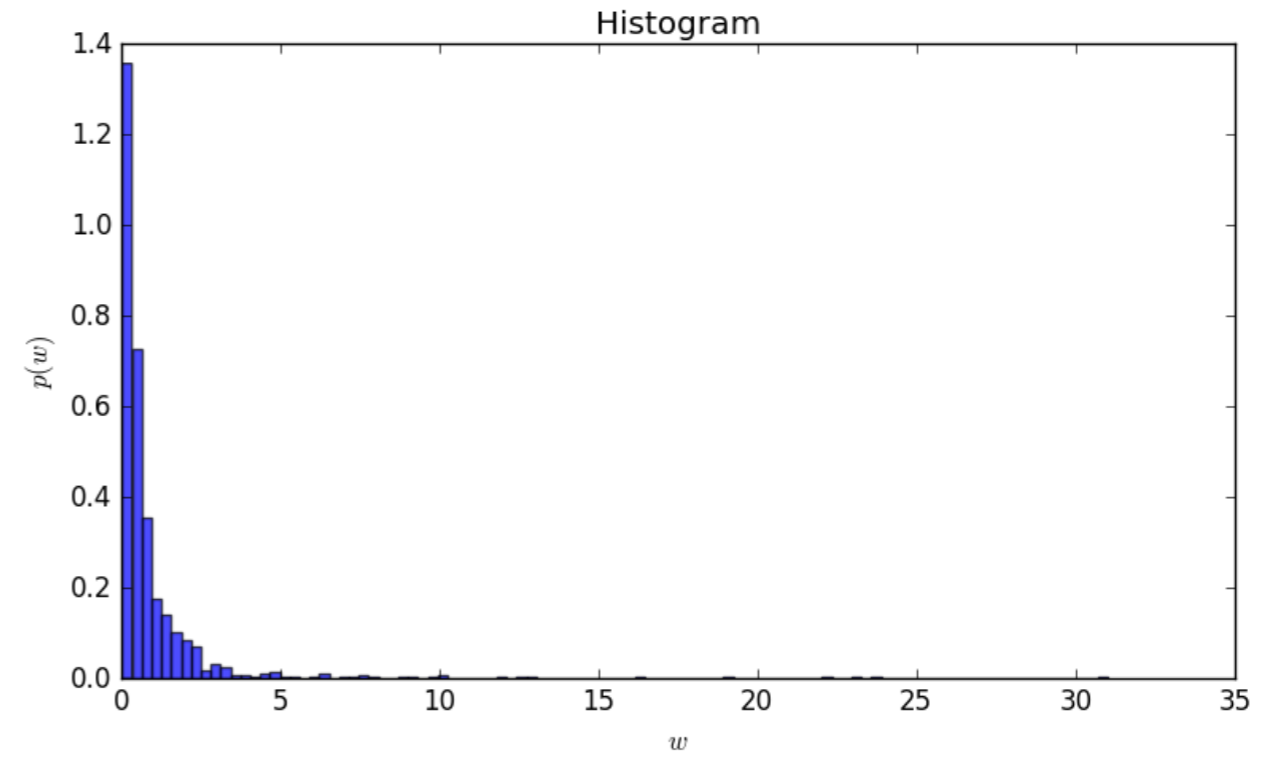
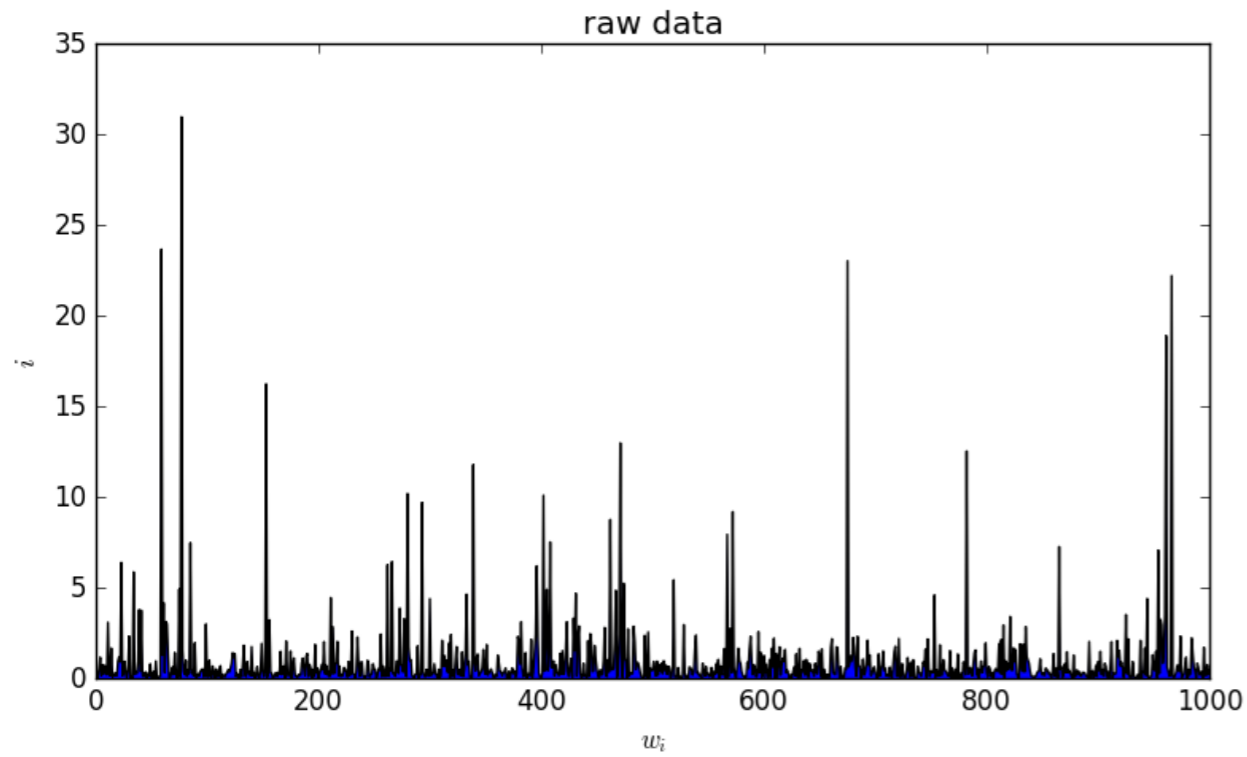
Unstable system (rich-gets-richer)

$$J_{ij} = J_{ji} < 0$$

Risk attitude (risk aversion $g < 1$, risk love $g > 1$)

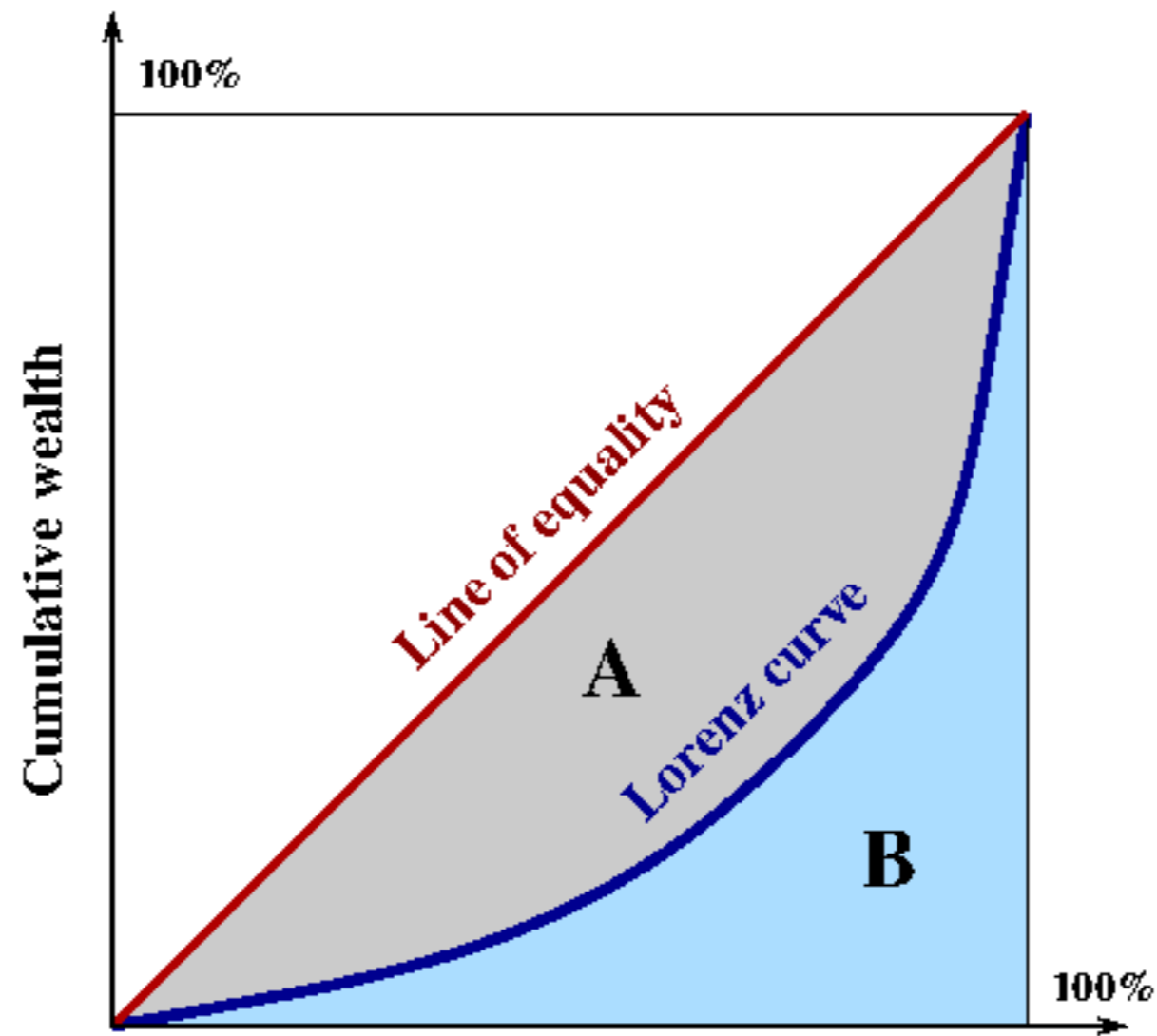
$$\max \left| \Delta W_{i\tau} \right| = g W_{i\tau}$$

From histograms to Lorenz curve



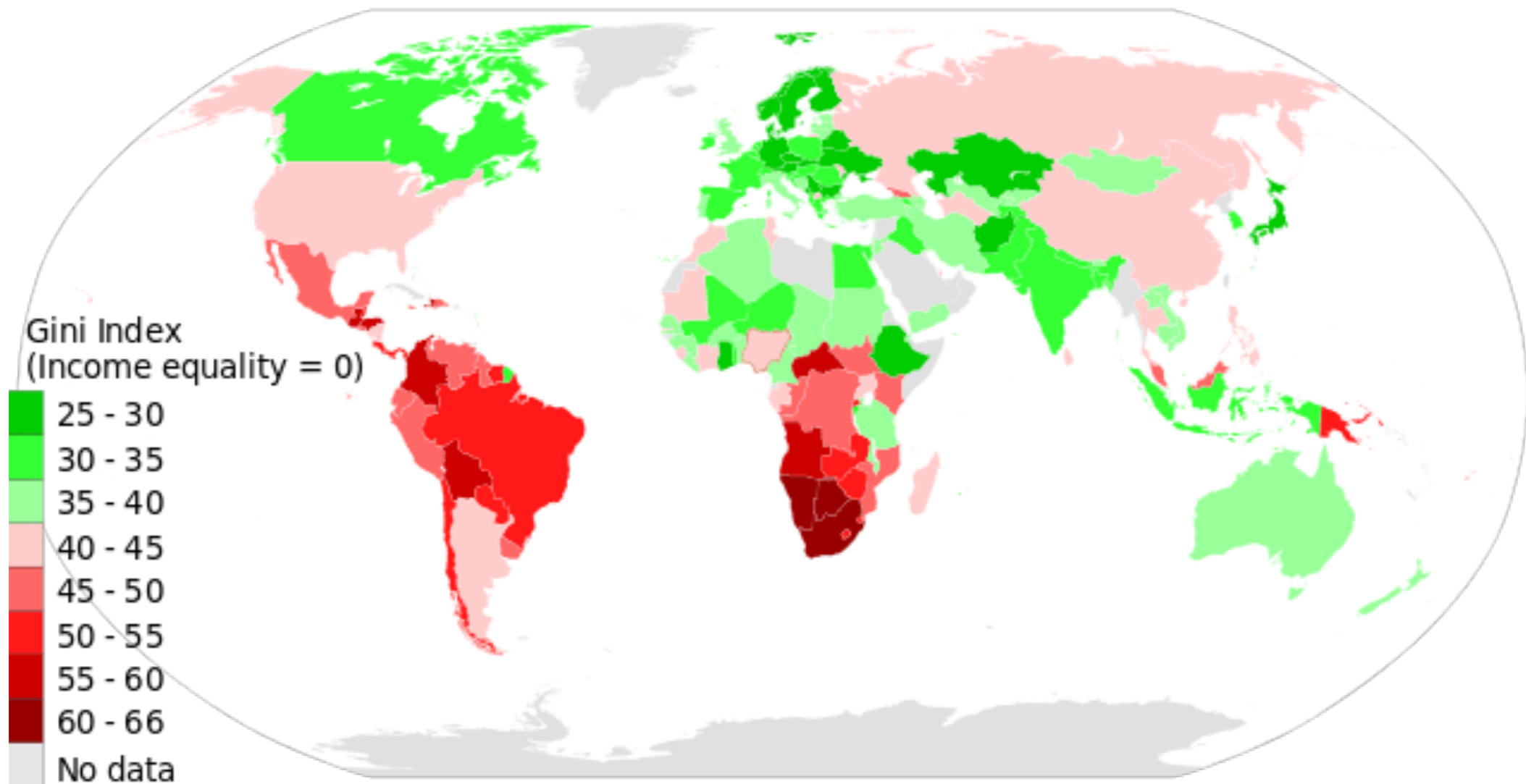
Lorenz curve and Gini coefficient

$$G = \frac{A}{A+B} \quad 0 \leq G \leq 1$$



Cumulative share of people from lowest to highest wealth

Gini coefficient by country



source: Wikipedia; by M. Tracy Hunter

Some results

- Mean-field
- Transaction rate $J = J_{ij}$
- Capital tax β
- Volatility $\sigma^2 = \text{Var}(\log \lambda_{i\tau})$
- Risk neutral

Continuous time limit

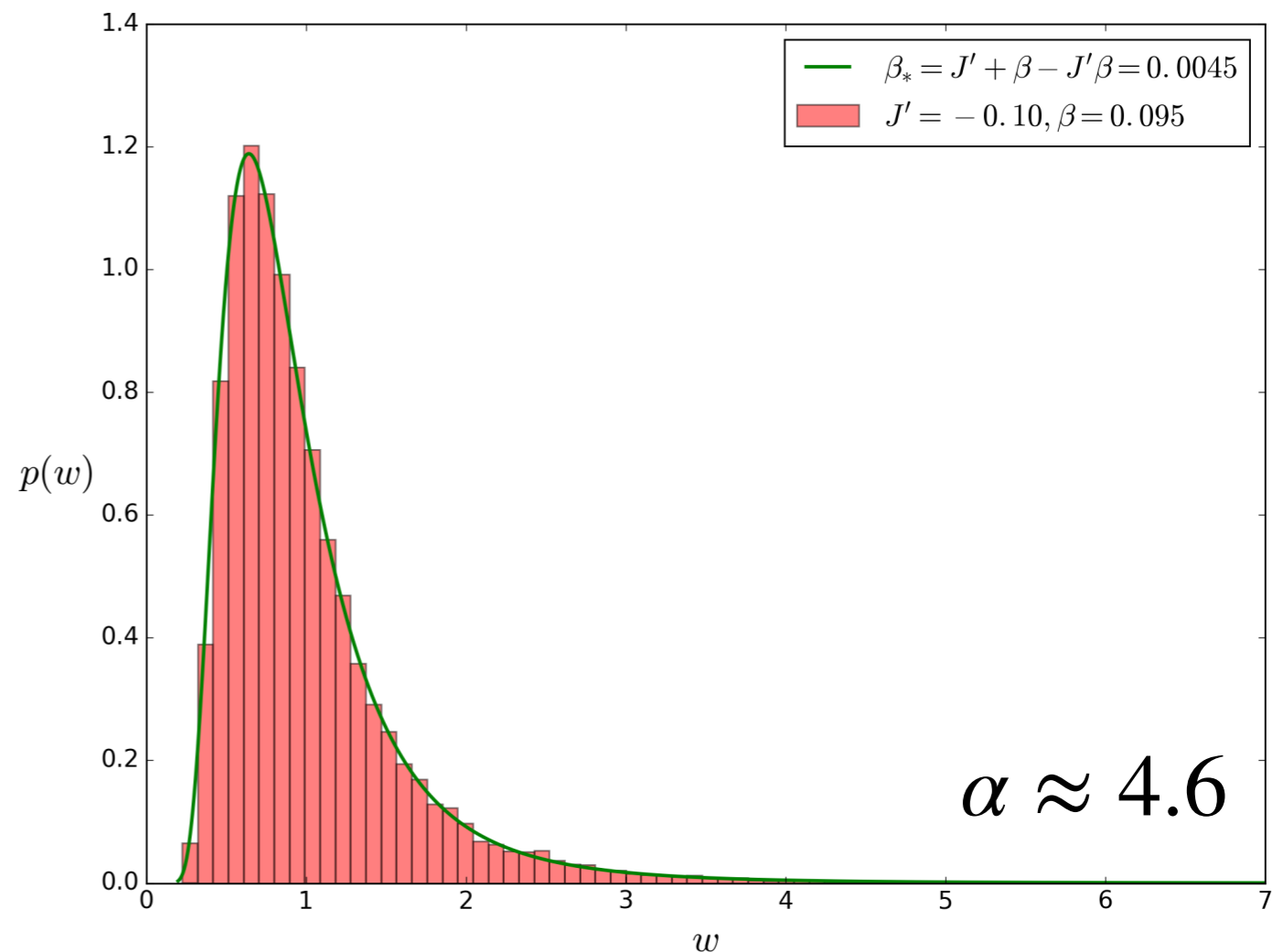
$$\frac{\partial}{\partial t} P(w, t) = \beta_* \frac{\partial}{\partial w} \left((w - 1)p(w, t) \right) + \frac{\sigma^2}{2} \frac{\partial^2}{\partial w^2} \left(w^2 p(w, t) \right)$$

$$\beta_* = \beta + J - \beta J > 0$$

Stationary state

$$P_{eq}(w) = \frac{c}{w^{1+\alpha}} \exp\left(-\frac{\alpha - 1}{w}\right)$$

$$\alpha = 1 + 2 \frac{\beta_*}{\sigma^2}$$

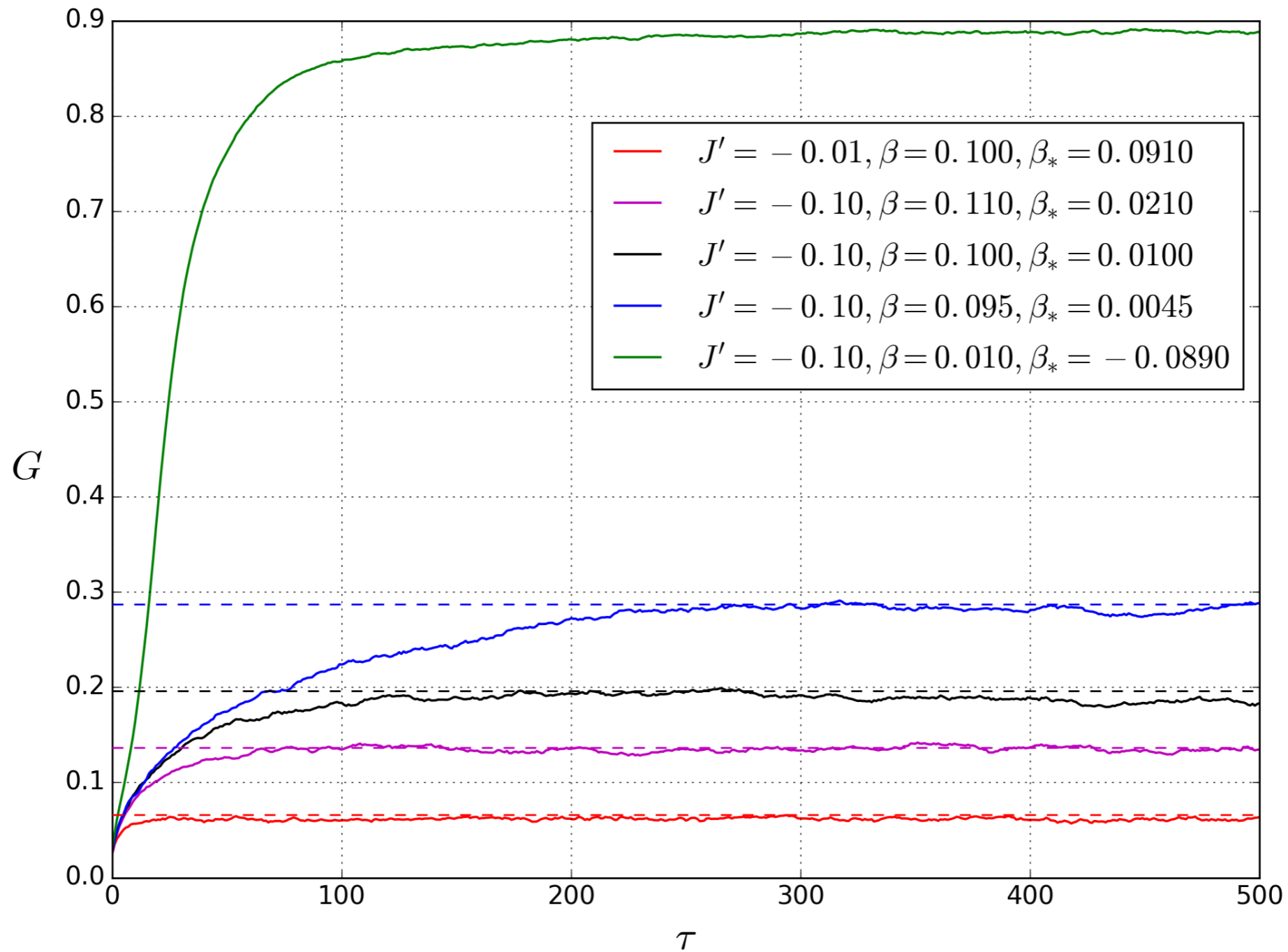


Rich-gets-richer

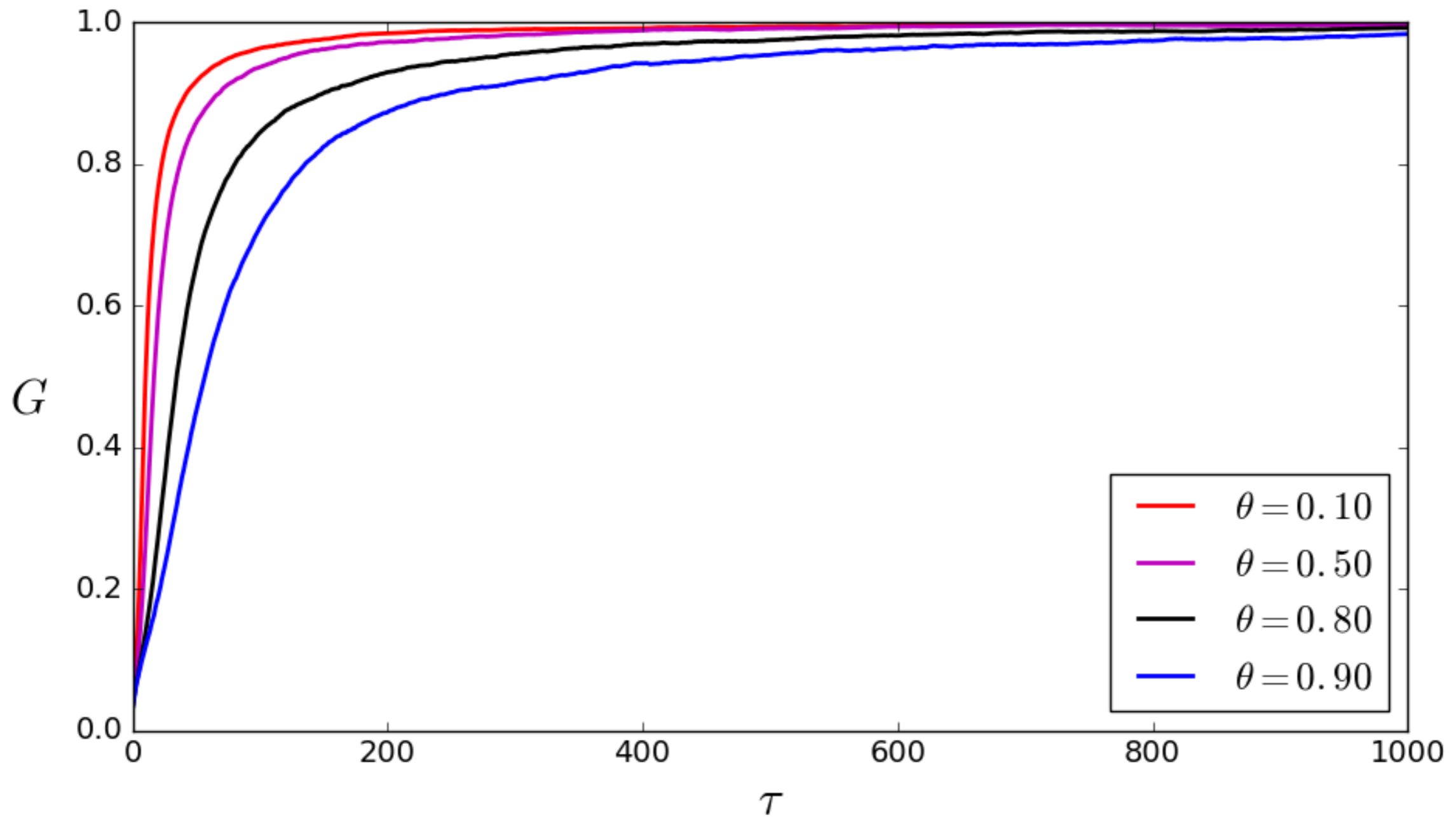
$$\beta_* = \beta + J - \beta J < 0$$

Accumulated advantage

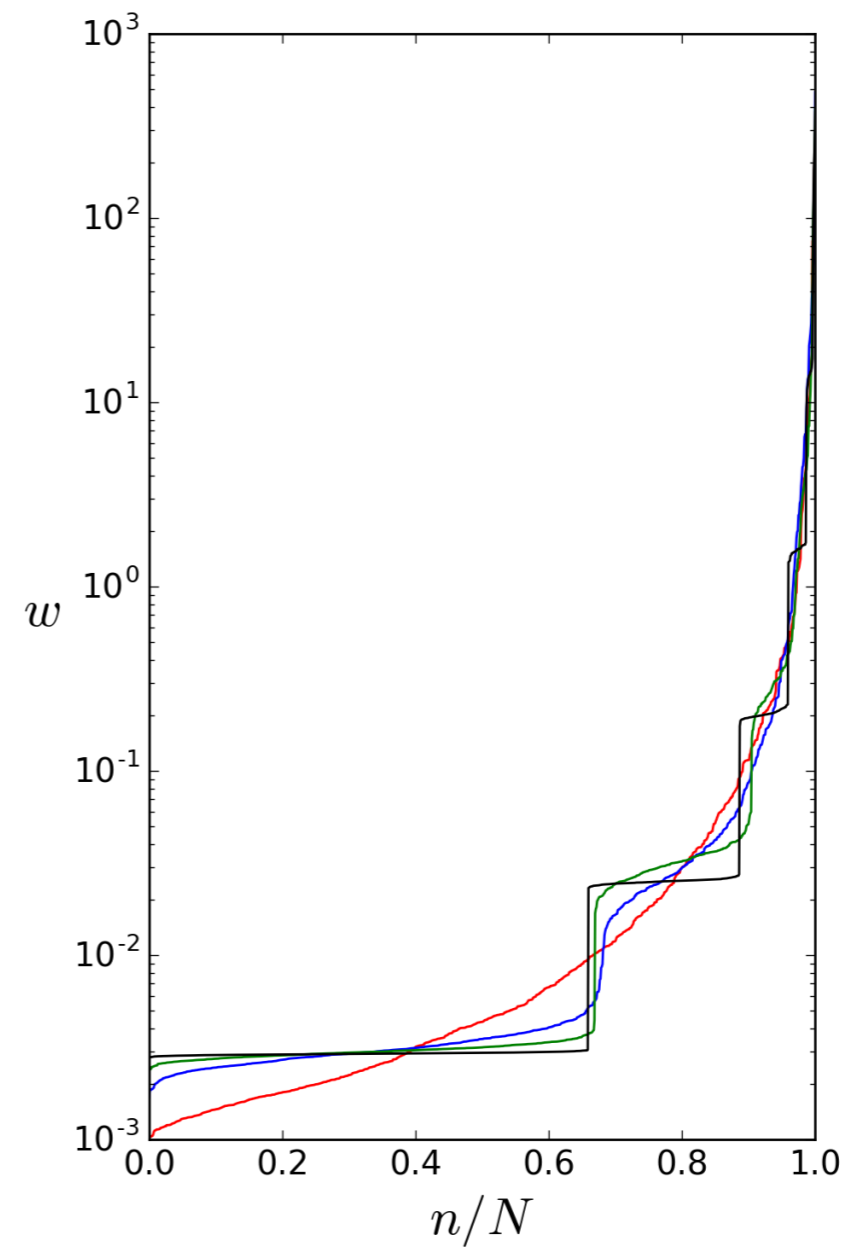
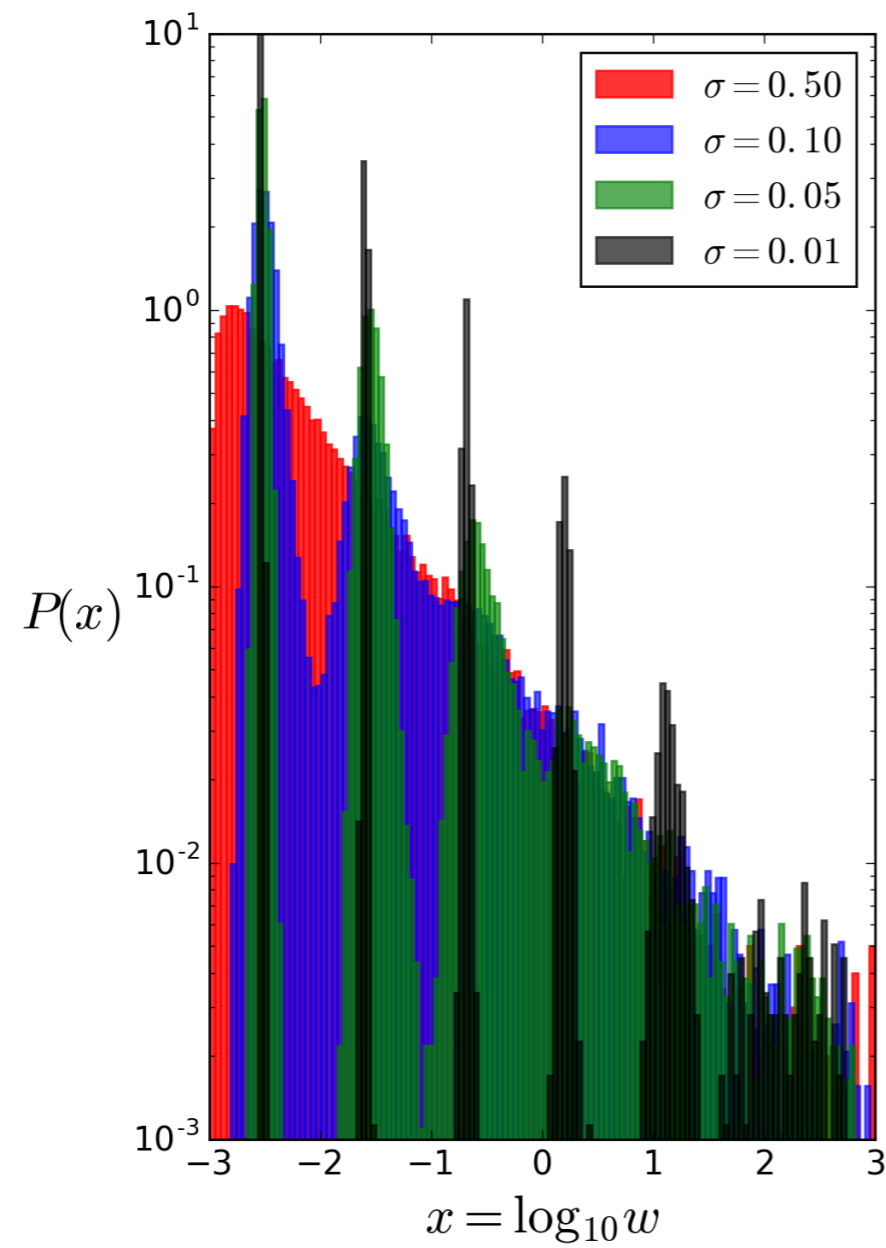
Matthew effect



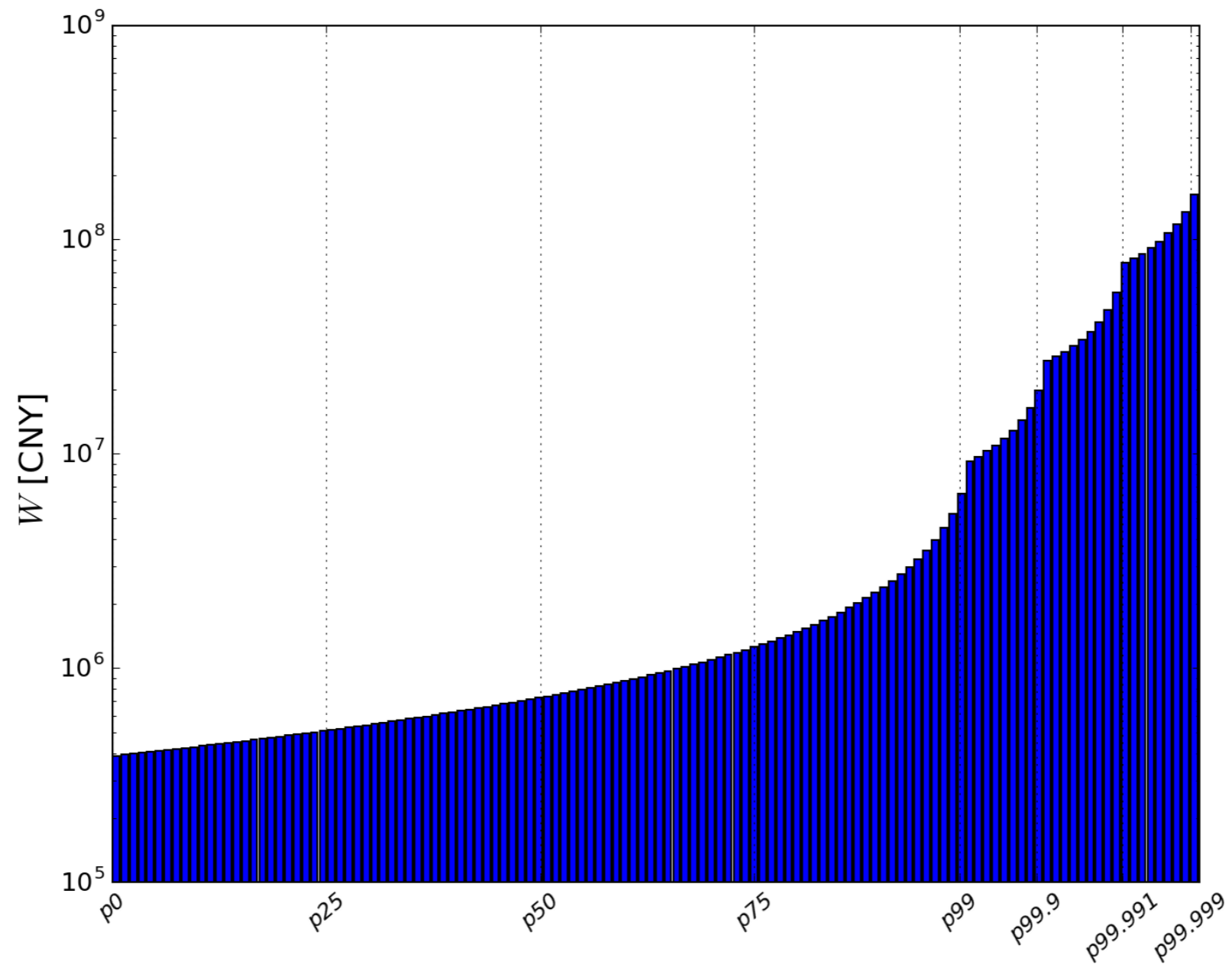
Evolution of Gini coefficient (income tax)



Separation of wealth classes

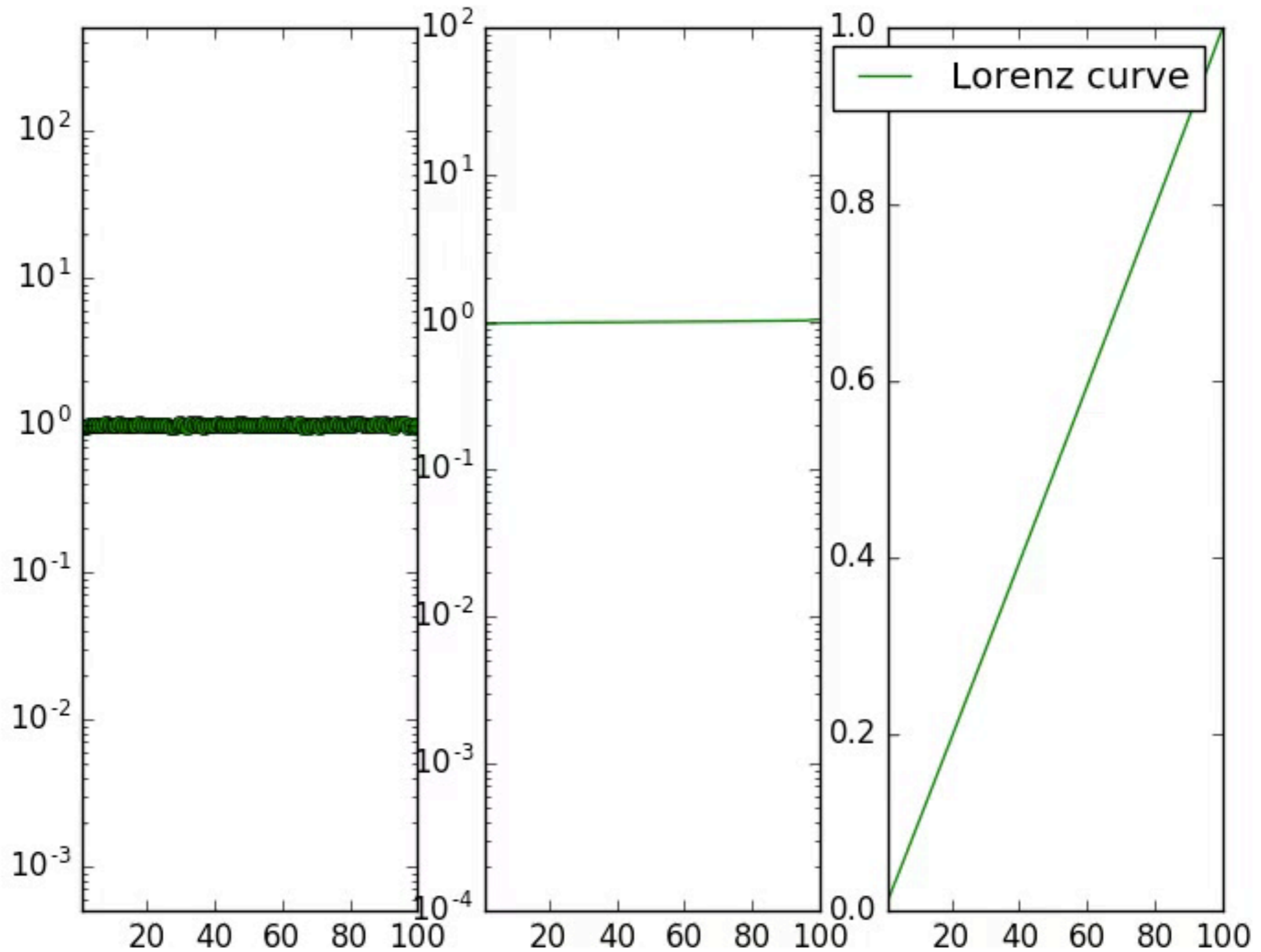


Distribution in urban areas of China

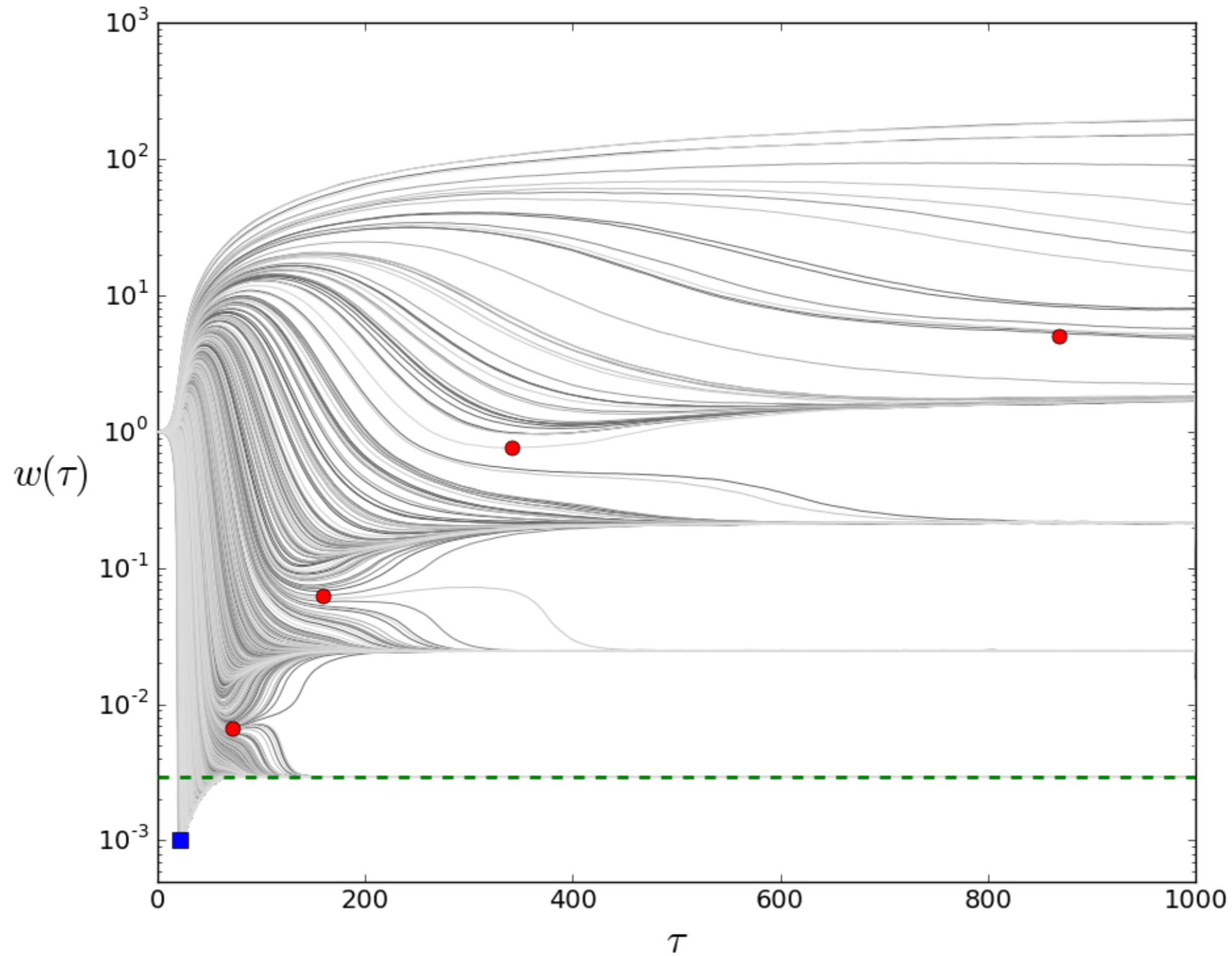


<http://wid.world/data>

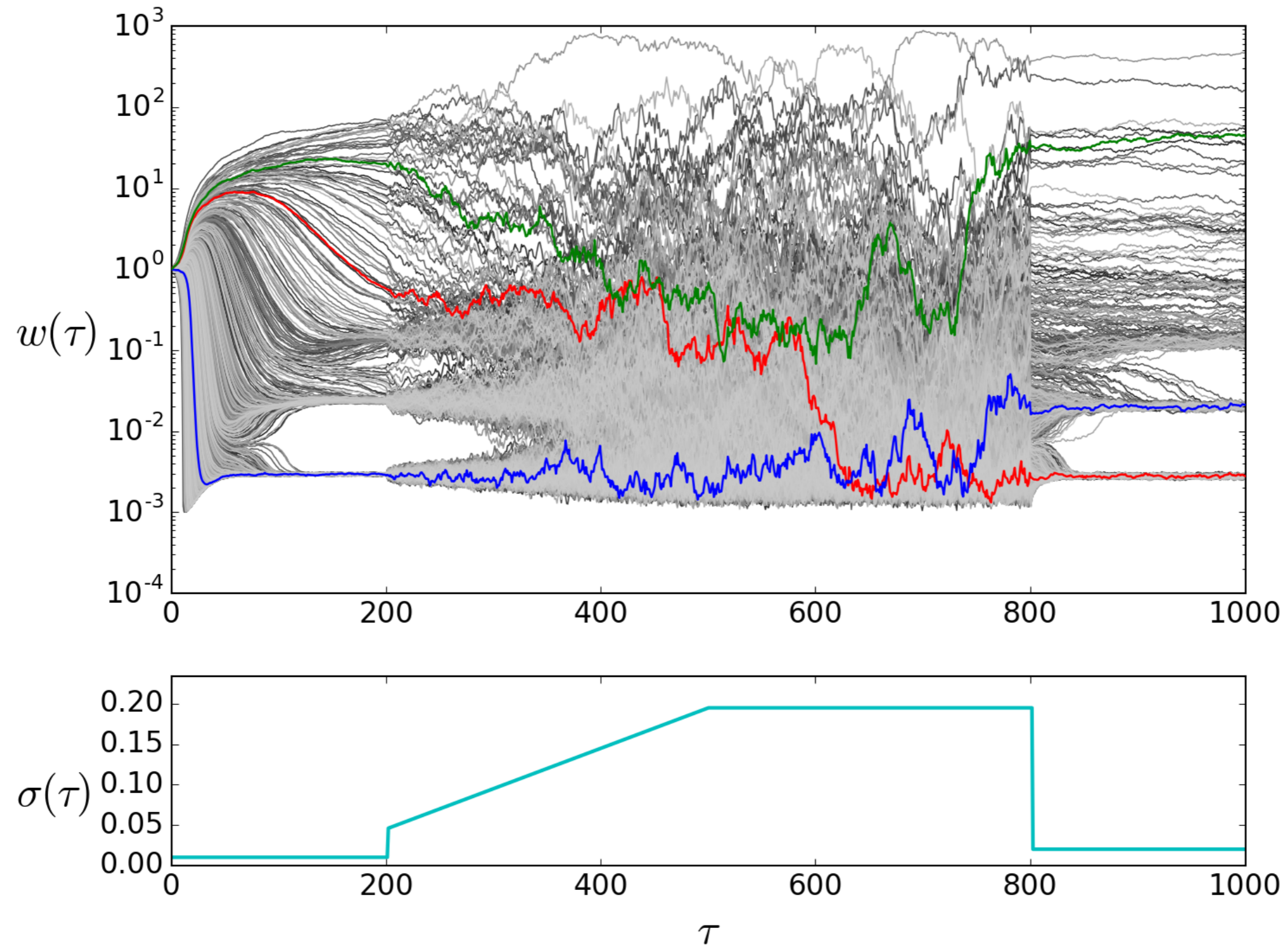
Time evolution



Evolution for extremely small volatility



Varying volatility



Summary

- agent model / emergent phenomena in macroeconomy
- model of aggressive market (rich gets richer)
- capital vs income tax
- social stratification (observed in data)

Thomas Piketty

" ... The inequality $r > g$ implies that wealth accumulated in the past grows more rapidly than output and wages. This inequality expresses a fundamental logical contradiction. ...

... The right solution is a progressive annual tax on capital. This will make it possible to avoid an endless inegalitarian spiral while preserving competition and incentives for new instances of primitive accumulation...

... The difficulty is that this solution, the progressive tax on capital, requires a high level of international cooperation and regional political integration. ..."

Thank you!

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