Appendix No. 3

Summary of professional accomplishments

1 Personal information

1. Name and surname: Tomasz Chwiej
2. Scientific titles:
   a) Master of Science, AGH University of Science and Technology, Cracow , 2001 year
   b) Doctor in Physics, Faculty of Physics and Applied Computer Sciences, AGH University of Science and Technology, Cracow, 2005 year
   Title of PhD thesis: *Theoretical research of artificial molecules and excitonic systems for a single and coupled quantum dots*
   thesis supervisor: prof. dr hab. Stanisław Bednarek
3. Former and current employment:
   a) years: 2005-2007, assistant, Faculty of Physics and Applied Computer Sciences, AGH University of Science and Technology, Cracow,
   b) years: 2007- , adjunct, Faculty of Physics and Applied Computer Sciences, AGH University of Science and Technology, Cracow,

2 Scientific achivement

Series of publications under the title: *Analysis of the localized and the conducting electron states in semiconductor quantum rings*

List of publications:

[R1] T. Chwiej, B. Szafran, *Pinning of electron densities in quantum rings by defects: Symmetry constraints and distribution of persistent currents*, Phys. Rev. B 79, 085305 (2009)

[R2] T. Chwiej, B. Szafran, *Few-electron artificial molecules formed by laterally coupled quantum rings*, Phys. Rev. B 78, 245306 (2008)

[R3] T. Chwiej, K. Kutorasiński, *Effect of Coulomb correlation on electron transport through a concentric quantum ring-quantum dot structure*, Phys. Rev. B 81,165321 (2010)

[R4] T. Chwiej, B. Szafran, *Fractional conductance oscillations in quantum rings: wave packet picture of transport in a few-electron system*, J.Phys.: Condens. Matter 25, 155802 (2013)

[R5] T. Chwiej, B. Szafran, *Schrödinger-Poisson calculations for scanning gate microscopy of quantum rings based on etched two-dimensional electron gas*, Phys. Rev. B 87, 085302 (2013)

[R6] T. Chwiej, B. Szafran, *Quantum ring conductance sensitivity to potential perturbation in an external magnetic field*, Phys. Rev. B 89, 195442 (2014)

Full texts of articles [R1–R6] are given in Appendix No. 6.

3 Brief review of scientific achievements

3.1 Introduction

At present, nanotechnology allows for creating with a high precision the semiconductor nanostructures of specific shapes and sizes. The main characteristic common feature of nanostructures i.e. quantum wells, dots, wires and rings is an appearence of the quantum size effect. Due to small sizes of nano-objects, energies of the confined charge carriers becomes quantized while their temporary positions are correlated as the result of electrostatic interaction and Pauli exclusion principle. Therefore a complete description of electronic system properties can be done with the use of quantum mechanics methods only. Quantization of energy levels results in resonance character of electron tunneling in quantum dots [1] while interaction between carriers is essential for Wigner molecules/crystal [2] creation and in fractional quantum Hall effect [3]. In quantum rings, due to their topology, there are observed especially strong quantum interference effects. In open rings which conduct the current, the wave function phase of a charged particle can change independently in both arms of the rings as a result of its interaction with vector potential (geometrical phase) or with electric field (dynamical phase) [4]. Superposition of both partial waves on exit of the ring makes the probability of electron transfer dependent on acquired phase difference. Thus, the change of magnetic or electric field may induce the conductance oscillations what is called the magnetostatic or the electrostatic Aharonov-Bohm effect [4]. The conductance oscillations may also appear as a result of interaction of magnetic moment with electric field what is observed in Aharonov-Casher effect [5]. A counterpart of Aharonov-Bohm effect in the closed quantum rings (which do not conduct the current) are energy oscillations driven by persitent currents induced in magnetic field [6]. Specific properties of quantum rings like easy manipulation of the amount of a confined charge in closed rings or control over the flow of a single charge carrier by application of quantum interference effects in open rings make their studies are not only significant for pure scientific point of view but are also important due to their potential applications as basic building blocks for the nanoelectronic devices. At present, quantum rings are used as quantum interferometers [7], detectors [8] and sources of microwave radiation [9]. Moreover, their applications in spintronics as spin filters [10] or quantum gates [11] are also studied. The main purpose of presented research [R1–R6] was to obtain the description of some chosen properties of localized and conducting electronic states in quantum rings. Properties of localized states are analyzed in works [R1, R2] and heavy stress was put on the analysis of experimentally measured quantities such as chemical potential and dipolar magnetic moment which are strongly depended on magnetic field strength. In remaining part of publications series [R3–R6] there is disscused the magnetoconductane of open rings for single-electron transport regime. There are considered two cases. In first case, transmitted electron interacts inelastically with other localized charges [R3, R4] while in second case it scatters elastically on static perturbation introduced intentionally to potential confinement of the ring [R5, R6]. In both cases particular emphasis was placed on the analysis of effects which are the results of interaction of a single electron or a few electrons system with magnetic field e.g.: normal and fractional Aharonov-Bohm effect, an action of magnetic forces and magnetic field influence on spatial localization of charge carriers. All works [R1–R6] are theoretical and problems raised there were solved numerically. The localized state i.e. not carrying the current, is understood here as the state of a single electron or a few electrons which wave function is completely localized in closed quantum ring (or in a system of coupled rings). Properties of localized states for $N = 1, 2, 3$ electrons are analyzed in works [R1, R2]. In first work [R1] there is disscused the effects of a confinement potential symmetry of single quantum ring on spatial distributions of charge and current densities, chemical potential and dipolar magnetic moment induced by orbital motion of electrons. Similar analysis was carried out for a system of two laterally coupled quantum rings for a weak, medium and strong tunnel coupling in work [R2]. Properties of conducting states for open quantum rings are discussed in works [R3–R6]. These rings are usually connected with at least two metallic gates which are the source and the drain that enable one to control the current flow in a system. Since electron emitted from the source may pass the open ring with finite probability (what makes the current flow between the source and drain) its state is the conducting one. Conductance oscillations observed in open quantum rings when magnetic field is changed result from magnetostatic Aharonov-Bohm effect while their amplitude depends on a number of magnetic flux quanta piercing the ring. Due to the interferential character of these oscillations they are very sensitive to any perturbation leading

to randomization of wave function phase of charged particle. Despite this fact, the mean free path in semiconductor quantum rings is usually much larger than their spatial sizes and therefore electron transport is ballistic [12, 13]. Large mean free path results from spatial separation of charge carriers and ionized atoms of dopants in nanostructure. Moreover, experiments with quantum rings are usually performed in very low temperature (e.g. $T < 1$ K). Then, the phase coherence time is then mainly limited by inelastic scattering processes like electron-phonon and electron-electron (dominant effect for homogeneous electron gas) interactions [14]. Even then, the phase coherence time may be larger than the time needed for electron to go through a ring what makes the electron transport coherent. However, when the Fermi electrons inelastically interact with other charges which are localized in the vicinity of the ring [16] or even in the ring (for low electron density) [17] then the amplitude or the period of Aharonov-Bohm oscillations may significantly change. This problem is considered in works [R3, R4]. In first work [R3], the effect of the Coulomb coupling between a ring and a charged quantum dot on the Aharonov-Bohm oscillations is analyzed while considerations contained in work [R4] mainly are focused on a fractional character of Aharonov-Bohm oscillations observed for charged few-electron quantum rings [17]. As the conductance of quantum ring strongly depends on geometry of confinement potential, due to a quantum interference effects, any perturbation introduced to this potential immediately influences on ring conductance. Its change depends on position, range and magnitude of potential perturbation. This property is utilized in the Scanning Gate Microscopy (SGM) [18]. In this method, the maps of conductance changes are gathered when the metallic biased tip moves a few tens of nanometers above the surface of the nanostructure. The most popular interpretation of such conductance maps links them with the Local Density of States (LDOS) [12, 19, 20]. In work [R5] there is presented the numerical model of an open quantum ring which works in SGM regime. This model takes into account the metallic gate without excessive simplifications what has enabled to compute the ring conductance for a potential landscape similar to a real one. Problem of LDOS reconstruction by the conductance maps gathered in SGM is also considered in work [R6] for a moderate as well as for a strong magnetic field when an electron transport is held in quantum Hall regime. Geometrical sizes of closed semiconductor quantum rings may span between several and few tens of nanometers. They are fabricated in self-organized growth process [21] or by wet etching [22]. Open quantum rings have much larger sizes that are usually of the order of a few hundred nanometers. They are formed by means of surface oxidization [23], etching [12] or gating a layered planar nanostructure. In most cases, motion of charge carriers in the growth direction in semiconductor quantum rings is limited to a very thin region of a quantum well formed at one or at two inverted heterojunctions e.g. GaAs/AlGaAs or InGaAs/GaAs. Due to the large energy excitations, motion of carriers in this direction is frozen (only ground state is filled) what separates it from their orbital motion in a plane of a ring. Therefore system can be described effectively within a two-dimensional model and such approach was applied in works [R1–R6].

3.2 Closed quantum rings

3.2.1 Dependence of chemical potential and dipolar magnetic moment on symmetry of the quantum ring for a few electrons system

The real quantum rings do not usually have strictly rotational symmetry due to imperfection of their fabrication methods. For self-organized quantum rings there are observed significant oscillations of their height what appears very often in the form of two maxima facing each other. In addition, these maxima may be correlated with the changes of width of the ring channel and local chemical stechiometry of semiconductor nanostructure compounds [24, 25] what translates on the depth of confinement potential. Etched rings may also have lower symmetry and it results from nonhomogeneous upper and/or side surface [12, 22] while in rings created with oxidization method the height and the width of a barrier formed under the trenches is not constant in the whole struture [16, 23]. In work [R1] there was solved the eigenproblem of $N = 1, 2, 3$ electrons confined in the quantum ring with perturbed rotational symmetry. Considered quantum ring is placed in homogeneous magnetic field directed perpendicularly to the surface of a nanostructure. In calculations there has been used a model confinement potential with geometrical and material parameters as for etched quantum ring [22]. In this model, the charge carriers are confined in vicinity of heterojunction made of $In_xGa_{1-x}As/GaAs$. There

have been analyzed [R1] an influence of quantum ring symmetry on some experimentally measured quantities like chemical potential and dipolar magnetic moment induced by orbital motion of electrons. Chemical potential in quantum rings can be determined by means of capacitance spectroscopy [26,27], magnetoconductance measurements in Coulomb blockade regime [23] or by absorption measurements of infrared radiation [26]. In work [R1], considerations concerning spin magnetic moment are skipped since its contribution to the resultant moment is up to a dozen or so times smaller than the orbital one induced in high magnetic field [24, 25, 28, 29]. Additionally, as the spin configuration is established for a certain range of magnetic field its contribution is fixed in contrast to orbital magnetic moment [30]. Energy of N-electrons ground state confined in quantum ring with rotational symmetry oscillates with period $\Delta B = B_0/N$ [6, 38], where: $B_0 = h/e/S$ is the basic Aharonov-Bohm oscillation period and S is the surface area of the ring. Due to angular momentum ($N \geqslant 1$) and spin configuration ($N \geqslant 2$) transformations induced in magnetic field, the energy maxima of N electrons system appear in the form of sharp kinks ($\bigwedge$) which are separated by the smooth minima of parabolic shape. In consequence, the chemical potential ($\mu_N = E_N - E_{N-1}$) has also noticeable sharp kinks of shapes: $\bigwedge$ and $\bigvee$, whereas the dipolar magnetic moment ($m_N = -\partial E_N/\partial B$) changes linearly bewteen the points where it is noncontinuous [R1]. When a single defect like shallow quantum dot is placed in a ring [R1] then the rotational symmetry is broken. Perturbation mixes closely lying, almost energetically degenerous states of different angular momentum but have the same spin configuration. In consequence, some energy crossings are replaced by anticrossings in energy spectrum. For the ground state, the sharp kinks in energy maxima ($\bigwedge$) which correspond to angular momentum transformations in magnetic field becomes smooth [R1] i.e. the derivative $\partial E_N/\partial B$ becomes continuous. For that reason, in quantum rings with single defect, chemical potential and dipolar magnetic moment of single electron are smooth functions independently of magnetic field strength. However, for two and three electrons, both $\mu_N(B)$ i $m_N(B)$ quantities become completely smooth only in quite high magnetic field ($B > 2.5$ T) when the system is permanetly spin polarized due to Zeeman effect [R1]. Moreover, the spin polarization makes also a disappearance of fractional oscillations. On the other hand, in lower magnetic field ($B < 2.5$ T), quantities: $m_2(B)$, $m_3(B)$ and $\mu_3(B)$ are almost identical to those obtained for the quantum ring with rotational symmetry. Only $\mu_2(B)$ has different magnetic field dependency since it remains smooth. If two identical quantum dots are put in quantum ring symmetrically in relation to its center then the rotational symmetry of the ring changes to a pointlike one. Few electrons states can be then divided in two groups i.e. odd and even states with respect to reflection $\boldsymbol{r} \rightarrow -\boldsymbol{r}$. It has been found [R1] that energies of ground states of single electron as well as of three electrons for point like symmetry change in magnetic field in the same manner as in unperturbed ring i.e. maxima of energy take the form of directed upward kinks ($\bigwedge$). Since, that type of kinks result from the crossings of energy levels corresponding to the states of different angular momentums or spin configurations, the dipolar magnetic moment changes in magnetic field similarly as for the ring with perfect rotational symmetry. But an energy and magnetic moment of two electrons confined in a ring with two defects have the same dispersion relation in magnetic field as for the ring with single defect. Both quantities change smoothly in high magnetic field ($B > 3$ T) [R1]. Thanks to such specific diversity of few electrons energies in magnetic field there appear a chance for determination of the confinement potential symmetry in quantum ring. For example, when two defects are placed in the ring then chemical potential has smooth maxima and minima for two and three electrons respectively in high magnetic field ($B > 3$ T) whereas they have kinks for a ring with rotational symmetry. But, chemical potential of single electron confined in a single-defected ring is completely smooth whereas for unperturbed and for double-defected rings its maxima have kinks ($\bigwedge$). This conclusion is one of the main results presented in work [R1]. In work [R1] there has been disscussed also behaviour of charge densities and persistent currents in magnetic field. If there is only a single defect in quantum ring then the charge density strongly oscillates in vicinty of anticrossings and forms local islands. The increase of magnetic field strength may even lead to a spatial separation of electrons due to their stronger electrostatic interaction. Thus, the Wigner molecule may come into existence. Even though, creation of two-electrons Wigner molecule becomes easier if two defects are put in the ring since it enhances their spatial localization, the three-electron Wigner molecule formation in the ring with two defects is prohibited due to restrictions imposed on spatial configuration of electronic system. When electrons are spatially separated then persistent current loop which originally moves along a

ring channel is broken. However, current does not completely disappear but it turns into current vortices which circulate around the density islands. These vortices have diamagnetic character what in consequence makes the direction of orbital magnetic moment to be antiparallel to that for external magnetic field one. Due to monotonous disappearance of current along the ring, dipolar magnetic moment induced by orbital motion of electrons becomes continuous function of magnetic field in the vicinity of anticrossings, for $N = 1$ always but for $N = 2, 3$ only in high magnetic field. Persistent current and charge density are much less susceptible to magnetic field changes in the vicinity of energy minimum. It was shown [R1] that reorientation of the magnetic moment which takes place for energy minimum becomes a continuous process in high magnetic field. As the quantum ring considered in work [R1] is two-dimensional, both paramagnetic and diamagnetic components of density current do not cancel each other as it is predicted for a strictly one-dimensional quantum ring. In fact they mutually compensate since they flow on internal and external sides of the ring channel respectively. Explanation of both, the reasons of charge density oscillations and the reasons of specific type of magnetic moment reorientation for a single quantum ring with potential defects is the second important result featured in work [R1].

3.2.2 Dependence of chemical potential and dipolar magnetic moment on tunnel coupling for two laterally coupled quantum rings.

Properties of localized states of a few electrons confined in two laterally coupled quatum rings are considered in work [R2]. Such systems of electrostatically and/or tunnel coupled quantum rings are usually fabricated during self-organized growth. During this process rings may create stochastic sets of rings [26,27,34], long chains or even two-dimensional lattices [31] and the strength of their coupling depends on the growth conditions. Considerations presented in work [R2] are based on the solutions of eigenproblem for $N = 1, 2, 3$ electrons confined in the system. In calculations, it has been used the same model confinement potential and method (exact diagonalization) as in work [R1]. It was shown that besides an electrostatic interaction between the localized charge carriers also the tunnel coupling between two rings has great influence on chemical potential and dipolar magnetic moment. If tunnel coupling is negligible due to a very wide barrier separating both rings, then energy spectrum of a single electron confined in a system is the same as for a single ring. Energy levels oscillates in magnetic field with Aharonov-Bohm period equal $\Delta B = B_0 = h/e/S$. If one ring is get closer to another one, the tunnel coupling is increased what results in a stronger localization of a single electron ground state in the vicinity of the barrier. In addition, the amplitudes of oscillations of ground state energy, chemical potential and orbital magnetic moment in magnetic field are diminished. Oscillations of these quantities dissapear when two rings are merged. Then all the charge density is localized in a joint area like in a single quantum dot. When two electrons are put in a system then both spatially separate from each other in ground state due to their repulsive Coulomb interaction. Each ring confines exactly one electron. Both the single electron wave functions are strongly modified by electrostatic interaction and do not overlap even for strong tunnel coupling. Therefore, the period of energy and magnetic moment oscillations in the ground state is the same as for a single electron. Moreover, it was shown that amplitudes of oscillations of these quantities are independent on the tunnel coupling [R2]. Since, ground states energies of one and two electrons oscillate with identical frequencies, the chemical potential $\mu_2(B)$ have only weakly marked minima of shape $\bigvee$ for weak tunnel coupling. Such shape results from the angular momentum transformations in a single electron ground state. These minima are replaced by smooth maxima when the tunnel coupling is strong enough to suppress energy oscillations in the ground state of single electron. An addition of third electron to the system makes the period of energy oscillations for $N = 3$ electrons two times smaller ($\Delta B = B_0/2$) than that for one or two electrons provided that the magnetic field is weak . Although, the growth of magnetic field intensity leads to a decrease in oscillations frequency, the period of energy oscillations always remains smaller than B_0. The pair correlation function calculated for three electrons has revealed that for a weak tunnel coupling there are two electrons localized in one ring and one electron localized in second ring [R2]. Therefore, the doubled frequency of the ground state energy oscillations is the result of spin transformation in doubly occupied ring for any direction of the third spin [R2]. However, when both rings are put so close to each other that the separating barrier disappears, then the extended molecular state streched over both rings is formed. In such situation, one of the electrons localizes

at the joint of both rings whereas other electrons move to the opposite edges minimaizing thus their mutual electrostatic interaction energy. In consequence, it leads to a significant drop in energy of odd parity spin polarized and odd parity spin nonpolarized states which oscillate in opposite phase around another low spin state of even parity. Similar arrangement of the lowest three energy levels oscillating in magnetic field was also predicted for a strongly elongated elliptical quantum dots [35]. It was also shown in [R2] that dependence of magnetic moment and especially dependence of chemical potential on magnetic field obtained for one and two electrons confined in two laterally coupled rings to a large extent depend on the magnitude of the magnitude of tunnel coupling. Thus, the dispersion relations of both quantities allows one to determine the magnitude of the tunnel coupling. On the other hand, it is not possible in the case of three electrons confined in two coupled rings since their spatial configuration in ground state is fixed. This conclusion is the main result in work [R2]. Another interesting outcome was obtained in [R2] for a two rings that have slightly different sizes. As it turned out the change of magnetic field strength may transfer some portion of the confined charge between the coupled quantum rings. Broken symmetry with respect to $\boldsymbol{r} \rightarrow -\boldsymbol{r}$ reflection lifts the energy levels degeneracy what diversifies single energy levels in separate rings. Now the ground state of single electron can be completely localized in one ring as it minimizes its confinement energy. As the magnetic field strength is increased (or decreased), the order of the two lowest energy levels corresponding to localization of electron in the bigger and in the smaller ring can be changed. In such case some part of the charge density flows between the rings. For $N > 1$ this mechanism is less efficient i.e. much smaller fraction of charge density may change its localization since the charge flow between the rings changes simultaneously value of interaction energy in a system. In such case, total amount of transferred charge is larger in the system that confines an odd number of electrons (e.g. for $N = 3$, $\Delta q_3 \approx 0.2$ e) than even one (e.g. for $N = 2$, $\Delta q_2 \approx 0.04$ e).

3.3 Open quantum rings

3.3.1 Electron transfer through a quantum ring capacitively coupled with quantum dot. Influence of Coulomb correlation on Aharonov-Bohm effect.

From weak localization theory appears that an electron-electron scattering is a main source of electron decoherence in homogeneous electron gas in low temperature. The phase coherence time limited by this process is usually much larger than the time needed for an electron to go through a ring and therefore decoherence effects are very often not included in considerations concerning single electron transport in quantum rings. However, when the charge density is decreased enough to make an interaction between charge carriers comparable with confinement energy or with the single electron energy excitations then the electronic correlation effects become extremely significant. For example, Meier et al. [41] has observed that amplitude of transconductance for a quantum ring changes when the charge flows through a quatum dot that was laterally coupled with the ring. Single electrons transport in the quantum ring can thus be hold in the Coulomb blockade regime [15, 23]. Also fractional periodicity of Aharonov-Bohm oscillations to be described in next section [6, 17, R4] is the effect of an electronic correlation. An extremely strong Coulomb correlation effect was observed by Mühle et al. [16] in experiment performed on open quantum ring that was capacitatively coupled with the second ring placed in its center. Surprisingly, measurements of magnetoconductance have revealed an existence of strong Aharonov-Bohm oscillations with frequency depended on geometry of closed ring. Similar problem was theoretically studied in work [R3] but for a single electron transfer through a two-terminal quantum ring capacitatively coupled with singly charged quantum dot placed in its center. Even though there is no tunnel coupling between them, motion of electron in a quantum ring and temporary charge density distribution in a dot are correlated due to the Coulomb interaction. For that reason, the time-dependent configuration interaction method was used for numerical simulation of this inelastic electron scattering problem. It has been shown [R3] that probability of electron transfer oscillates in magnetic field due to Aharonov-Bohm effect but amplitude of these oscillations strongly depends on polarization of internal quantum dot. Negatively charged dot creates a potential barrier which is too high to overcome for the low energy part of electron wave packet that moves in open quantum ring. Consequently, negatively charged dot decreases probability of electron transfer. On the other hand, if quantum dot is positively charged, then the transmitted electron feels a lower confinement potential

in the ring. It does not increase the backscattering but significantly accelerates electron motion in a ring and thus reduces the time it needs to get out from the ring. Therefore, probability of electron transfer in such case may be even two times larger than for a ring coupled with negatively charged dot [R3]. An influence of Coulomb coupling on Aharonov-Bohm oscillations in considered quantum dot-quantum ring nanostructure is to a large extent dependent on relation between interaction energy and excitation energies of particle that is localized in a dot. Since the kinetic part of a single-particle Hamiltonian depends inversly on its effective mass, the heavier particle is the less energy is needed to excite it. Consequently, heavy particle reacts more intensely to presence of an electron in quantum ring than the light one. When electron approaches the ring, the charge density localized in dot may start to oscillate due to the Coulomb interaction. If magnetic field is weak, denisty may oscillate along the axis established by the external channels have been attached to the ring. It results from the fact that confinement potential is symmetrical and probabilities of excitation for the lowest two states that have opposite values of angular momentum are very similar. Such oscillatory motion of charge density introduces specific changes in confinement potential of quantum ring. In consequence there appear an additional phase shift in wave function of transferred electron due to the electrostatic Aharonov-Bohm effect [32]. It has been found however, that these density oscillations are of marginal significance for amplitude and for frequency of Aharonov-Bohm oscillations. Dynamical phase shifts introduced in both arms of the ring are the same and thus can not change the phase difference on exit from the ring. An appearance of charge oscillations in a system is tightly connected with the magnitude of energy transferred from ring to dot. Such inelastic character of electron scattering in the ring makes the phase difference of both partial waves, that meet on its exit, to be partly randomized due to the dynamical change of the wave vector of transmitted electron. For that reason, probability of electron transfer through the ring does not completely disappear for magnetic flux $\phi = (n + 1/2)h/e,\ n = 0, 1, 2, \ldots$ as in the case without Coulomb interaction. On the other hand, frequency of Aharonov-Bohm oscillations does not change if density oscillations have one-dimensional character and there is no density redistribution along the radial direction in the ring. It has been also shown [R3] that the amount of energy transferred to the dot is tightly connected with the period of Aharonov-Bohm oscillations as it depends on value of electron's transfer probability. Maximum energy is given to the dot when probability of electron transfer through the ring is minimal and vice versa.
Description of Coulomb correlation influence on Aharonov-Bohm oscillations in the quantum dot-quantum ring structure is the main result in work [R3].
Effect of Coulomb coupling on the amplitude of Aharonov-Bohm oscillations was experimentally observed for a system of two capacitatively coupled concentric quantum rings [16]. Measurements of magnetoconductance have showed that the amplitude of oscillations induced by motion of electrons in internal (closed) quantum ring is much larger than that resulting from thier motion in open (larger) ring. The first attempt to explain this discrepancy was done in work [16]. According to it the amplitude of oscillations for larger ring is strongly reduced due to additional scattering of Fermi electrons on defects. It does not explain however, why the amplitude of oscillations for closed ring was not suppressed too. In view of the studies presented in work [R3] there can be another explanation of such specific behaviour of magnetoconductance. The excitation energies in quantum ring are much smaller than in quantum dot. Therefore, electrons confined in internal quantum ring may absorb an energy transferred from open quantum ring more effectively. When an electron goes through the open ring it looses part of its energy what diminishes the amplitude of Aharonov-Bohm oscillations for larger ring [R3] but simultaneously makes the oscillations for smaller (closed) ring more distinct. Consequently, the more energy is given to internal ring, the more decreased is the amplitude of Aharonov-Bohm oscillations for larger ring whereas more enhanced is that for the smaller one.

3.3.2 Fractional periodicity of Aharonov-Bohm oscillations for a singly and for a doubly charged quantum ring

An appearance of fractional periodicity Aharonov-Bohm oscilliations for open quantum rings was predicted by Chakraborty et al. in work [6]. The ground state energy of a few electrons confined in quantum ring oscillates periodically when magnetic field is constantly changed. However, the period of these oscillations may depend not only on the geometry of the ring but also on the number of confined electrons as $\Delta B = \Delta B_0/N$, where: $\Delta B_0 = (h/e/S)$ is the period of oscillations for normal i.e. integral

Aharonov-Bohm effect while S is the surface area of the ring. As there is a close relation between energy oscillations in closed quantum ring and conductance oscillations in open ring, it was suggested that the fractional periodicity might be also detected in magnetoconductance measurements. Although, this hypothesis was proved later experimentally [17], the fractional periodicity of oscillations is not always clearly visible in conductance [12, 23]. As indicated in work [6], appearance of fractional oscillations is mainly conditional on existence of exchange interaction in fermionic system strong enough to lift the spin degeneracy for the lowest excited energy states [6, 38, 39]. Problem with an appearance of fractional oscillations in magnetoconductance for open quatum ring was raised in work [R4]. For this puropose there was solved an electron scattering problem for two-terminal open quantum ring that confines initially one or two electrons. The model confinement potential used in calculations has contained material and geometrical parameters as for the ring fabricated by oxidization method with carriers confined in GaAs/AlGaAs heterojunction region [17]. Additionaly, calculations were done with assumptions that electron injected to the input channel moves in its lowest energy subband and the motion energy of electron is small enough ($\sim$ 1meV) to prevent other electrons from leaving the ring what might happen as a result of their mutual electrostatic interaction. As the electronic correlation plays a key role in considered inelastic scattering problem, the time-dependent configuration interaction method was used for simulations of time evolution of a few electrons wave function and in calculations of electron transmission coefficent. It has been shown [R4] that fractional periodicity of Aharonov-Bohm oscillations is particularly strong when the ring confines initially only one electron. Then the period of oscillations equals $\Delta B = \Delta B_0/2$. These oscillations appear independently from the mutual orientations of spins of interacting electrons as well as from magnitude of lateral confinement energy in both arms of the ring. Calculations were performed for lateral confinement as for quantum harmonic oscillator with energy levels spacings $\hbar\omega = 2.4,\ 4.0,\ 8.0$ meV that correspond to effective channel widths $W = 87.1,\ 67.5,\ 47.7$ nm respectively. The increase of lateral confinement energy makes the effect of fractional periodicity of Aharonov-Bohm oscillations to be more distinct what manifests in growth of their amplitude. However, when electron scatters in the ring that initially confines two electrons, the expected period of oscillations i.e. $\Delta B = \Delta B_0/3$ was found only for the strongest lateral confinement $\hbar\omega = 8$ meV ($W = 47.7$ nm). The increase of lateral confinement makes the spatial localization of electrons in this direction more stronger. Then, the energies of Coulomb and exchange interactions in quantum ring are enhanced as well what is the main condition for an appearance of fractional periodicity of Aharonov-Bohm oscillations. Thus, the results presented in [R4] are in accordance with the conclusions contained in works for closed quantum rings [6, 38] and for partly open rings [39]. Interestingly, different results have been obtained by Kotimäki and Räsänen [40] for small quantum ring that had single lead attached to it. Based on outcomes got for a rings of radius $r = 10, 20$ nm, they have claimed that electronic correlation has less impact on fractional periodicity of Aharonov-Bohm oscillations than the geometrical changes of confinement potential. However, the confinement energies for such small rings much exceeds the electrostatic interaction in the system what make the motion of electrons to be uncorrelated. For that reason, Aharonov-Bohm oscillations observed in [40] did not have expected $\Delta B = \Delta B_0/N$ dependency as obtained for larger rings (in [R4] the radius of the ring equals 132 nm). Besides the studies on fractional oscillations for unperturbed quantum ring, an influence of potential perturbation on this effect was also investigated. If a single defect like shallow quantum dot is placed in one arm of the ring, it may double conductance oscillations frequency even for a single electron (i.e. not interacting with other electrons in the ring) due to Altshuler-Aronov-Spivak effect [37]. Such potential perturbation does not influence on the fractional periodicity in two-electrons systems provided that spin-singlet and spin-triplet configurations may be get with equal probability. It was found that fractional oscillations disappears when two electrons are initially confined in the ring. Broken symmetry of confinement potential combined with Coulomb repulsion leads to a spatial separation of electrons. Then the electron density is pinned in the vicinity of potential defect. Such spatial configuration of two electrons confined in the ring becomes stiff and only weakly sensitive to perturbation introduced by third electron that enters the ring. Therefore, even for such weak potential perturbation, oscillations with a period $\Delta B = \Delta B_0/3$ do not appear in a conductance.

In summary, the main achievements included in work [R4] was working out an advanced numerical model that allows to calculate the probability of electron transfer through the charged quantum ring with electronic correlation fully accounted. Within that model it has been explicitly demonstrated

that there exists a close relation between an amplitude of fractional Aharonov-Bohm oscillations and the magnitude of electron-electron interaction in the ring.

3.4 Computer simulations of an electron transport in the scanning gate microscopy

3.4.1 Solution of Poisson-Schrödinger problem for quantum ring in a scanning gate microscopy

The uniqe topology of quantum rings makes their conductane very sensitive to perturbation of confinement potential. For perturbed potential, the trajectories of electrons are changed what may even suppress Aharonov-Bohm oscillations [12, 13, 16]. This property is utilized in the scanning gate microscopy [12, 18, 44]. In this method, electrons interact with a ring potential that is intentionally perturbed by biased metalic gate put close to the electron gas. If the spatial size of perturbation is comparable with the wave length for electrons from Fermi surface then one may expect that changes in magnetoconductance should be correlated with Local Density of States (LDOS) for Fermi energy due to the Kramers-Kronig relation [19]. In the beginning, the maps of conductance changes induced by scanning gate were calculated with application of a static perturbation potental [12, 19] that had of Gaussian or Lorentzian shape. Actually, such potential perturbation has Lorentzian shape in a very large nanostructure only, what was confirmed by Gildemeister et al. [43] in experiment performed for a large quantum dot. However, when an examined nanostructure has smaller dimensions, the potential perturbation used in calculations should have take into account also how large is the screening effect caused by electrons confined in nanostructure as well as changes in gate polarization when the gate is moved close to the ring edges [12, 14, 19]. Method which enables one to model, in a more realistic manner, the potential perturbation in electron gas that fills a quantum ring, is presented in work [R5]. There has been used the numerical model based on the search of self-consistent solutions of Poisson-Schrödinger problem. Preliminary calculations were performed for a two-terminal quantum ring that was etched from InP/InAlAs/InGaAs/InAlAs layered nanostructure [12]. In this system, the donor atoms are placed inside the blocking barrier (InAlAs) about 15 nanometers below the surface. Electrons released by the ionized donors are partly transferred to the quantum well that constitutes the ring as well as to both external leads while the rest fill the surface states. An electron gas which conducts the current is buried about 25 nanometers below surface at the InGaAs/InAlAs heterointerface. Solutions of Poissson equation that have been found for this three-dimensional model of open quantum ring and scanning gate suspended above its surface have taken into account the following effects: 1) screening of the gate potential by an electron gas and surface electrons, 2) the charge induced at surfaces of semiconductor nanostructure and of metallic gate i.e. the effect of noncontinuity of dielectric constant, 3) finite spatial sizes of the ring what influences on the magnitude of potential screening and polarization of nanostructure and gate surfaces. Conductance of the ring was determined [R5] according to Landauer formula [49] while the transmission coefficients for subsequent conducting channels that appear in this formula were obtained from the solutions of time-independent and effectively two-dimensional Schrödinger equation which has taken into account intersubband scattering [45]. It is worth to notice that conductane can also be calculated in an alternative way by application of the Retarded Green Function defined for Schrödinger equation [42]. However this method is less efficient in the case of numerical computations made for spatial meshes with large number of nodes as for the problems considered in works [46, R5, R6]. Results presented in work [R5] show that electrons gathered temporarily in open ring and on its surface to a large extent react to the scanning gate potential. For that reason, both charge densities are locally redistributed so as to maximally screen the potential perturbation. Magnitudes of charge redistribution in the ring and at surface depend on gate position. When it flows directly above the ring, the electrons trapped on semiconductor surface give significantly larger contribution to the screening effect than electrons confined in a quantum well. It results from a smaller distance between the gate and semiconductor surface. Situation is reversed when the scanning gate is moved behind the ring edges. Then, electrons that fill the ring, screen the potential of the gate more strongly than those localized at ring surface [R5]. Such behaviour is mainly the effect of parametrization of phenomenological model used in calculations for describing density of states on InAlAs surface [47]. There were used two sets of

parameters in work [R5]. For first parametrization, as much as 63% of electrons released from donor's layer are localized on semiconductor surface while for second parametrization this factor drops to 40%. Division of free electron charge between the surface and the ring is important not only due to the difference in screening of gate potential by surface charge and electron gas but also due to its large impact on Fermi energy in the system. For I and II parametrizations, the Fermi energies were equal $E_{F1} \approx 45.5$meV and $E_{F2} \approx 76.1$meV respectively. On the other hand, the authors of experiment [12] have claimed that in the considered nanostructure all electrons can move to the quantum well but not to a surface. For the same donor density $n_d = 2 \times 10^{12}/\mathrm{cm}^2$ and similar geometry of nanostructure, they estimated the Fermi energy to be about $E_F = 100$meV [19]. Even though they did not explain the reasons of this charge imbalance, it probably results from specific arrangement of the donor layer. In considered nanostructure [12, R5], the donor layer which is the source of electrons is placed in InAlAs barrier region that separates a quantum well (ring) from a surface. Free electrons move to the well first due to the lower confinement energy. Next, they fill the localized states on surface due to a Fermi level pinning in energy gap of InAlAs [47]. Besides the stechiometry of InGaAs/InAlAs heterojunction, division of electrons amongst these two regions is conditional on the relative distance of the donor layer to each of them. Since positively charged donors significantly lowers the confinement potential in the vicinity of their layer, even a small shift of this layer in the growth direction may considerably change spatial distribution of electron density. That seems to be the most probable reason of Fermi energy difference in works [R5] and [12] what in consequence influences on both the potential landscape in nanostructure and on screening of the scanning gate. Results of numerical calculations [R5] show that the potential perturbation induced by scanning gate in the electron gas has the Lorentzian shape provided that is hung directly above the center of the ring channel. For a fixed gate bias, the increase of donors density makes an amplitude of potential perturbation smaller but it does not change its range. The spatial range of potential perturbation is comparable with the distance between the quantum ring and the surface of the gate what agrees well with the results obtained previously for a simpler model of gate potential [46]. As the gate is moved above the ring area or in its proximity then the conductance undergo the characteristic changes [R5]. When it flows directly above the ring and it is negatively polarized then the changes in conductance maps appear in the form of radial fringes with alternately lying regions of increased and decreased values. Such characterstic changes in conductance maps were previously observed in experiment by Hackens et al. [12]. Their appearance is caused by scattering of Fermi electrons on the repulsive potential hill induced by scanning gate inside a ring. However, this radial pattern is transformed to the concentric one if the gate is shifted behind an area of a ring [12, R5]. These concentric changes in conductance maps occur independently of gate polarization [R5] what is a manifestation of the electrostatic Aharonov-Bohm effect [32]. As the scanning gate is placed outside the ring, it perturbs the confinement potential in proximity of ring edges what may modify the phase difference of interferring partial waves on its exit. The phase difference is fixed if the gate is moved along the left or the right arm at fixed distance from a ring while it changes oscillating when this distance is increased [18]. These results were well reproduced within the Poisson-Schrödinger numerical model but not for the model Lorentz-shaped potential of the gate [48, R5]. It has been shown [R5] that the latter approximation discribes quite well the effective potential of the gate only if it is maximally screened by electron gas. It results from the fact that such approximation can not describe the long-distance Coulombic character of gate potential when its screening is much weaker. Therefore its application in numerical studies should be limited to the internal area of the ring. It has been also shown in [R5] that for inhomogeneous density distribution of ionized donors, being the source of electrons in the ring, the radial and the concentric changes in conductance maps are retained but the overall conductance pattern is only slightly distorted azimuthally.

In summary, the main purpose of work [R5] was to present the numerical model that allows to describe the changes in conductance of two-terminal quantum ring caused by scanning gate flowing above its surface. It is the main achievement appearing in work [R5]. Correctness of this model was confirmed by qualitative agreement between the numerical and experimental data.

3.4.2 Magnetic field effect on a conductance of quantum ring in scanning gate microscopy

The studies on the correspondence between the conductance changes and the local density of states (LDOS) for a two-terminal quantum ring in the scanning gate microscopy were continued in work [R6]. For this purpose, the correlation factor for both kinds of maps i.e. for conductance and for LDOS was calculated. Its large values unambiguously indicate that such correspondence exists while for small values does not. This quantity was analyzed in dependence on Fermi energy and magnetic field strength. In calculations there were used two-dimensional confinement potential of the quantum ring. The Fermi energy was changed in the range $E_F = 6 - 18$meV and therefore only four the lowest subbands were populated in two external channels attached to the ring and gave contributions to the conductance. Since LDOS and in consequence a correlation factor can be determined only in the ring, considerations were limited to this region exclusively. It also justifies the use of model potential of Lorentzian shape [12, R5] for approximation of perturbation potential which is introduced to electron gas by a scanning gate. Results of calculations presented in [R6] show that an appearance of Aharonov-Bohm oscillations in conductance of unperturbed ring strongly depends on both, the Fermi energy and the magnetic field $[G(E_F, B)]$, only if magnetic field is relatively weak ($B \approx 0.5$T) while vanish for strong fields ($B \approx 1$T). Disappearance of Aharonov-Bohm oscillations is related to activity of magnetic force in considered system. This force deflects the trajectories of electrons that enter the ring. In strong field, electrons are directed mainly to one of two arms of the ring. Effect of such injection imbalance of carriers is greatly enhanced for smoother ring-channels junctions. It has been also shown [R6] that the changes of correlation factor and changes of the first derivative of the conductance over the magnetic field $[\partial G(E_F, B)/\partial B]$ calculated for unperturbed ring have similar dependence on E_F and B. It means that for specific values of E_F and B, for which the conductance of unperturbed ring is highly sensitive to the magnetic field change, LDOS can be potenitally reconstructed by means of conductance map gathered in the scanning gate microscopy. However, perturbation potential must have a short-distance character in such case i.e. its spatial range can not be larger than a few nanometers what is a prerequisite for LDOS reconstruction. In other case, results are rather ambiguous. For example, the outcomes of calculations performed for a scanning gate with much larger effective spatial range of potential perturbation ($\sigma_L = 30$ nm) indicate that the conductance may significantly change when the gate is moved in proximity of the ring-channel junction. Unfortunately, these changes of ring conductance have turned out to be averaged over the distance several times larger than spatial oscillations of LDOS for Fermi electrons. In such case the conductance maps can not be used for LDOS reconstruction due to their small spatial resolution. As was shown in works [46, R5] the spatial range of potential perturbation induced by gate depends on the distance between the gate and the electron gas layer. In typical semiconductor nanostructure studied in scanning gate microscopy, this layer is situated at heterointerface (e. g. InGaAs/AlGaAs in work [12]) and buried at least 25 nanometers under the surface of the nanostructure. For the gate suspended 25 nanometers above the surface it gives an effective range of perturbation of about 50 nanometers. It can be decreased further by shifting the electron gas layer closer to the surface and moving the scanning gate slightly closer to the surface. Nonetheless, one may expect it would be hardly to get the spatial range of perturbation smaller than 30 nanometers in practice. It has ben shown in [R6] that reconstruction of LDOS in the scanning gate microscopy can be hardly done also if magnetic field is very strong. Electron transport is maintained then in quantum Hall regime. Due to suppression of backscattering for edge states that exclusively give contributions to conductance of the ring [44], the function that describes the dependence of ring conductance on the Fermi energy and on the magnetic field gets flat and forms plateau. Suppression of conductance oscillations in strong magnetic field becomes particularly distinct if the input and the output joints in the ring are getting smooth i. e. radius of their curvature becomes comparable with Fermi electron wave length. Reconstruction of LDOS for the points lying on the plateau is in most cases impossible [R6]. The only exception to this rule was found for abrupt ring and external leads joints geometry. For particular energies there occur very strong coupling between the wave function for an electron moving in external channel and the states localized at the entrance to and on exit from the ring. Since the states localized at the channel-ring joint have no angular momentum, they do not participate in electron transport. In consequence, activation of these resonance states decreases the conductance of a ring. Coupling of localized states with external channel is highly sensitive to the

potential perturbation induced by the scanning gate. As it was shown in work [R6], determination of LDOS for such resonances is possible provided that, the spatial extent of potential perturbation is small. Nonadiabatic character of changes in wave function of an electron resulting from an abrupt change in the confinement potential at channel-ring joint may also induce the conductance oscillations on the plateu. These oscillations appear due to formation of quasiclassical skipping electron orbit which falls entirely into ring arm. When an electron moves in the ring simultaneously bouncing from an external edge of the ring channel, it may bypass an exit from the ring what diminishes conductance value. Similar effect was noticed in experiment by Hackens et al. [14].
In summary, it has been demonstrated in work [R6] that the derivative of conductance over magnetic field for unperturbed quantum ring changes in similar manner as the correlation factor calculated for the LDOS and the conductance of the ring which is measured in the scanning gate microscopy. This is the main conclusion included in work [R6]. Since it unambiguously indicates the work points on the conductace map that allow to determine LDOS in a quantum ring, it may have potentially some impact on how the SGM experiments will be carried in a future.

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Bibliography

[1] W. G. van der Wiel, S. De Franceschi, J. M. Elzerman, T. Fujisawa, S. Tarucha, L. P. Kouwenhoven, Rev. Mod. Phys. 75, 1 (2002)
[2] J. Yoon,C. C. Li, D. Shahar, D. C. Tsui, M. Shayegan, Phys. Rev. Lett. 82, 1774(1999)
[3] H. L. Stormer, D. C. Tsui, A. C. Gossard, Rev. Mod. Phys. 71, S298 (1999)
[4] Y. Aharonov, D. Bohm, Phys. Rev. 115, 485(1959)
[5] Y. Aharonov, A. Casher, Phys. Rev. Lett. 53, 319(1984); J. Nitta, T. Bergsten, New J. Phys. 9, 341(2007)
[6] K. Niemelä, P. Pietiläinen, P. Hyvönen, T. Chakraborty, Europhys. Lett. 36, 533(1996)
[7] I. Neder, N.Ofek, Y. Chung, M, Heiblum, D. Mahalu, V. Umansky, Nature 448, 333(2007)
[8] J.-H. Lee, J.-H. Dai, C.-F. Chan, S.-C. Lee, IEEE Photon. Technol. Lett. 21, 721(2009)
[9] S. Park, S.-S. Kim, L. Wang, S.-T. Ho, IEEE J. Quant. Electron. 41, 351 (2005)
[10] Diego Frustaglia, Martina Hentschel, Klaus Richter, Phys. Rev. Lett. 87, 256602 (2001); M. Lee, C. Bruder, Phys. Rev. B 73, 085315 (2006)
[11] P. Földi, B. Molnár, M. G. Benedict, F. M. Peeters, Phys. Rev. B 71, 033309 (2005)
[12] B. Hackens, F. Martins, T. Ouisse, H. Sellier, S. Bollaert, X. Wallart, A. Cappy, J. Chevrier, V. Bayot, S. Huant, Nature Physics 2, 826(2006); F. Martins, B. Hackens, M. G. Pala, T. Ouisse, H. Sellier, X. Wallart, S. Bollaert, A. Cappy, J. Chevrier, V. Bayot, S. Huant, Phys. Rev. Lett. 99, 136807 (2007)
[13] D.-I. Chang, G. L. Khym, K. Kang, Y. Chung, H.-J. Lee1, M. Seo, M. Heiblum, D. Mahalu, V. Umansky, Nature Physics 4, 205 (2008)
[14] B. Hackens, F. Delfosse, S. Faniel, C. Gustin, H. Boutry, X. Wallart, S. Bollaert, A. Cappy, and V. Bayot, Phys. Rev. B 66, 241305(R) (2002)
[15] A. Mühle, W. Wegscheider, R. J. Haug, Appl. Phys. Lett. 92, 013126(2008)
[16] A. Mühle, W. Wegscheider, R. J. Haug, Appl. Phys. Lett. 91 , 133116 (2007)
[17] U. F. Keyser, C. Fühner, S. Borck, R. J. Haug, M. Bichler, G. Abstreiter, and W. Wegscheider,Phys. Rev. Lett. 90, 196601(2003)
[18] H. Sellier, B. Hackens, M. G. Pala, F. Martins, S. Baltazar, X. Wallart, L. Desplanque., V. Bayot, S. Huant, Semicond. Sci. Technol. 26 064008 (2011)
[19] M. G. Pala, B. Hackens, F. Martins, H. Sellier, V. Bayot, S. Huant, T. Ouisse, Phys. Rev. B 77, 125310 (2008)
[20] M. Mendoza, P. A. Schulz, Phys. Rev. B 68, 205302 (2003); M. Mendoza, P. A. Schulz, Phys. Rev. B 71, 245303 (2005)
[21] T. Mano,T. Kuroda, S. Sanguinetti, T. Ochiai, T. Tateno, J. Kim, T. Noda, M. Kawabe, K. Sakoda, G. Kido, N. Koguchi, Nano Lett. 5, 425 (2005)
[22] M. Bayer, M. Korkusinski, P. Hawrylak, T. Gutbrod, M. Michel, and A. Forchel, Phys. Rev. Lett. 90, 186801(2003)

[23] A. Fuhrer, S. Lüscher, T. Ihn, T. Heinzel, K. Ensslin, W. Wegscheider, M. Bichler, Nature 413, 822 (2001)
[24] N. A. J. M. Kleemans, I. M. A. Bominaar-Silkens, V. M. Fomin, V. N. Gladilin, D. Granados, A. G. Taboada, J. M. Garcia, P. Offermans, U. Zeitler, P. C. M. Christianen, J. C. Maan, J. T. Devreese, and P. M. Koenraad, Phys. Rev. Lett. 99, 146808 (2007)
[25] V. M. Fomin, V. N. Gladilin, S. N. Klimin, J. T. Devreese, N. A. J. M. Kleemans, and P. M. Koenraad, Phys. Rev. B 76, 235320 (2007)
[26] Axel Lorke, R. Johannes Luyken, Alexander O. Govorov, Jörg P. Kotthaus, J. M. Garcia, and P. M. Petroff, Phys. Rev. Lett. 84, 2223 (2000)
[27] W. Lei, C. Notthoff, A. Lorke, D. Reuter and A. D. Wieck, Appl. Phys. Lett. 96, 033111 (2010)
[28] V. M. Fomin, V. N. Gladilin, J. T. Devreese, N. A. J. M. Kleemans, and P. M. Koenraad, Phys. Rev. B 77, 205326 (2008)
[29] Bartłomiej Szafran, Phys. Rev B 77, 205313(2008)
[30] G.Bardsen, E. Tölö, A. Harju, Phys. Rev. B 80, 205308(2009)
[31] J. Wu, Z. M. Wang, K. Holmes, E. Marega Jr., Z. Zhou, H. Li, Y. I. Mazur, G. J. Salamo, Appl. Phys. Lett. 100, 203117 (2012)
[32] S. Washburn, H. Schmid, D. Kern, R. A. Webb,Phys. Rev. Lett. 59, 1791 (1987); B. Krafft, A. Förster, A. van der Hart, Th. Schäpers, Physica E 9, 635(2001)
[33] J. Planelles, F. Rajadell, J. I. Climente, Nanotechnology 18, 375402(2007)
[34] A. Lorke, R. Blossey, J.M. Garcia, M. Bichler, G. Abstreiter, Materials Science and Engineering B88, 225 (2002)
[35] Y. Li, C. Yannouleas, U. Landman,Phys. Rev. B 76, 245310(2007)
[36] P. Mohanty, E. M. Q. Jariwala, R. A. Webb, Phys. Rev. Lett. 78, 3366 (1997)
[37] B. L. Altshuler, A. G. Aronov, B. Z. Spivak, JETP Lett. 33, 94(1981)
[38] Y. Hancock, J. Suorsa, E. Tölö, A. Harju, Phys. Rev. B 77, 155103 (2008)
[39] M. Ijäs, A. Harju, Phys. Rev. B 85, 235120 (2012)
[40] V. Kotimäki, E. Räsänen, Phys. Rev. B 81, 245316 (2010)
[41] L. Meier, A. Fuhrer, T. Ihn, K. Ensslin, W. Wegscheider, M. Bichler, Phys. Rev. B 69, 241302(R) (2004)
[42] G. Metalidis, P. Bruno, Phys. Rev. B 72, 235304(2005); F. Sols, M. Macucci, U. Ravaioli, K. Hess, J. Appl. Phys. 66, 3892(1989)
[43] A. E. Gildemeister, T. Ihn, M. Sigrist, K. Ensslin, D. C. Driscoll, and A. C. Gossard, Phys. Rev. B 75, 195338 (2007)
[44] B. Hackens, F. Martins, S. Faniel, C.A. Dutu, H. Sellier, S. Huant, M. Pala, L. Desplanque, X. Wallart, V. Bayot, Nature Communications 1, 39 (2010)
[45] M. R. Poniedziałek, B. Szafran, J. Phys.: Condens. Matter 22, 215801(2010)
[46] B. Szafran, Phys. Rev. B 84, 075336(2011)
[47] J. S. Hwang, G. S. Chang, W. C. Hwang, W. J. Chen, J. Appl. Phys. 89, 396(2001)
[48] M.D. Petrović, F.M. Peeters, A. Chaves, G.A. Farias, J.Phys.:Conens. Matter 25, 495301(2013)
[49] R. Landauer, IBM J. Res. Develop. 1, 223 (1957)