Prescaling and Far-from-Equilibrium Hydrodynamics in the Quark-Gluon Plasma

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AM and Jürgen Berges, Phys. Rev. Lett. 122, 122301 (2019)

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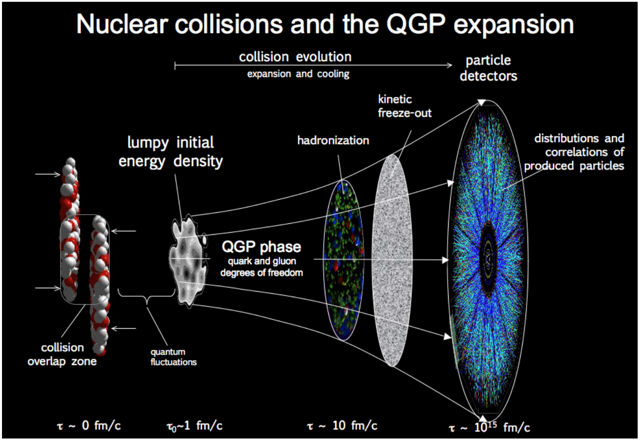
Isolated quantum systems and universality in extreme conditions

Motivation

Relativistic heavy ion collisions

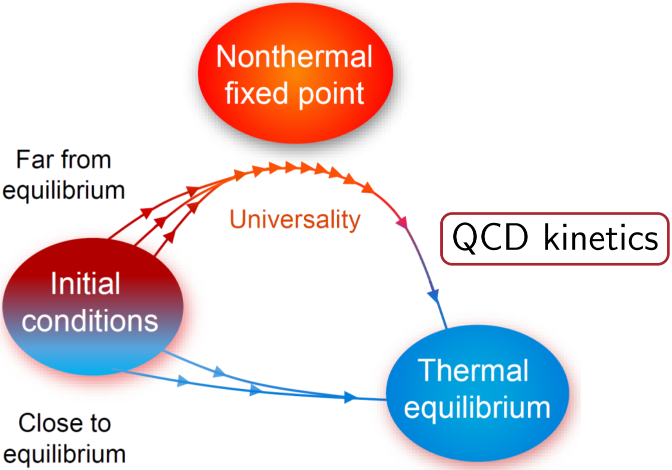
Heavy ion programs at RHIC (since 2000) and LHC (since 2010).



Sorensen, Quark-gluon plasma 4, 2010

Information loss $\Longrightarrow$ macroscopic (hydrodynamic) description of QCD.

$\alpha_s \to 0$, overlap with classical fields, early times



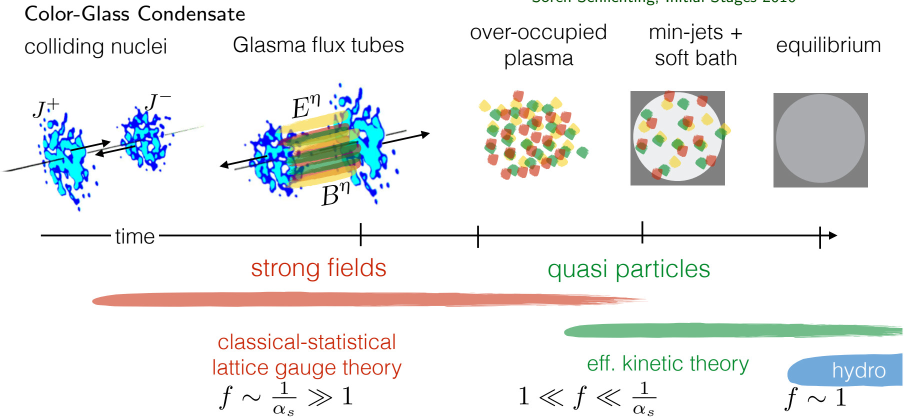
$\alpha_s \approx 0.3$, overlap with hydrodynamics, late times

Equilibration in heavy ion collisions: weak coupling picture

At high energies and densities — asymptotic freedom $\alpha_s \ll 1$

Gross, Wilczek; Politzer (1973)[1, 2]

Sören Schlichting, Initial Stages 2016

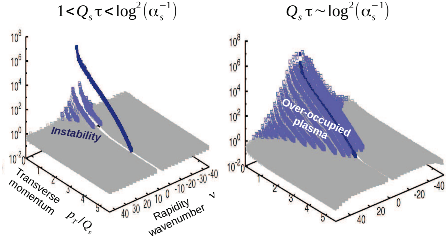


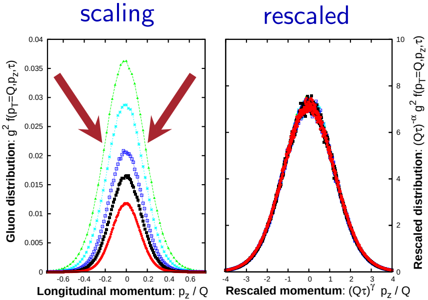
QCD kinetic theory — bridge between early and late time dynamics.

Non-thermal fixed point (NTFP) for gauge theories

For $f \sim A^2 \gg 1$ classical-statistical Yang-Mills describes gluon evolution

Aarts, Berges (2002), Mueller, Son (2004), Jeon (2005)





Berges, Schenke, Schlichting, Venugopalan (2014) [3] Berges, Boguslavski, Schlichting, Venugopalan (2014) [4]

Self-similar scaling $\Longrightarrow$ loss of information

$$f_g(p_\perp, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_\perp, \tau^{\gamma} p_z), \quad \tau = \sqrt{t^2 - z^2}$$

Universal exponents: $\alpha \approx -\frac{2}{3}, \quad \beta \approx 0, \quad \gamma \approx \frac{1}{3}$

Scaling phenomena also seen in scalar theories, cold atom experiments

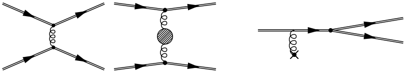
Orioli et al. (2015) [5], Mikheev et al. (2018) [6], Prüfer et al. (2018) [7], Erne et al. (2018) [8]

QCD effective kinetic theory

Weakly coupled quark and gluon quasi-particles in a soft background.

Arnold, Moore, Yaffe (2003)[9]

$$\partial_\tau f_{g,q} \underbrace{- \frac{p_z}{\tau}\partial_{p_z} f_{g,q}}_{\text{expansion}} = - \underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]}$$



Complete leading order description:

- elastic $2 \leftrightarrow 2$ scatterings: $gg \leftrightarrow gg$, $qq \leftrightarrow qq$, $gq \leftrightarrow gq$, $gg \leftrightarrow q\bar{q}$
- particle number changing $1 \leftrightarrow 2$ processes: $g \leftrightarrow gg$, $q \leftrightarrow qg$, $g \leftrightarrow q\bar{q}$ (includes interference effects — LPM suppression)
- only parameter — the coupling constant α_s.

"Bottom-up" thermalization scenario

Baier, Mueller, Schiff, and Son (2001)[10]

I) over-occupied	$p_z \sim \frac{Q_s}{(Q_s\tau)^{1/3}}$	$1 \ll Q_s\tau \ll \alpha_s^{-3/2}$
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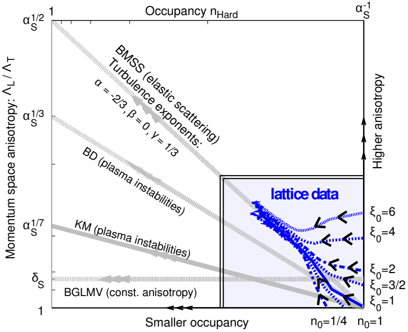
From classical simulations to kinetic theory

Initial distribution $f_0 \sim \frac{1}{g^2}\theta(Q_s - \sqrt{p_\perp^2 + \xi^2 p_z^2})$

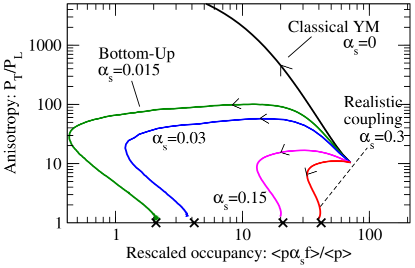
classical-statistical Yang-Mills

kinetic theory of gluons

Anisotropy



Berges, Boguslavski, Schlichting, Venugopalan (2014)[11]



Kurkela and Zhu (2015)[12]

Occupancy

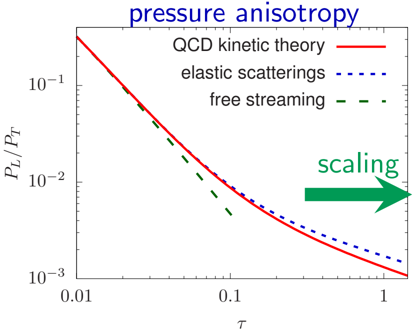
Far-from-equilibrium dynamics with QCD kinetic theory

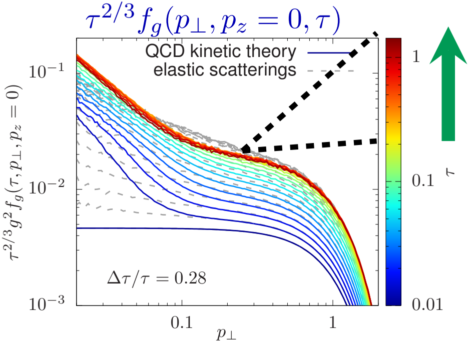
Scaling in leading order QCD kinetic theory with fermions

Initial conditions $f_g = \frac{\sigma_0}{g^2} e^{-(p_\perp^2 + \xi^2 p_z^2)}$, $\quad \sigma_0 = 0.1, \quad g = 10^{-3}, \quad \xi = 2$

Scaling regime is reached at late times

$$f_g(p_\perp, p_z, \tau) = \tau^{-2/3} f_S(p_\perp, \tau^{1/3} p_z), \quad \tau \to \tau/\tau_{\text{ref}}$$





Non-thermal fixed point reached in full QCD kinetic evolution.

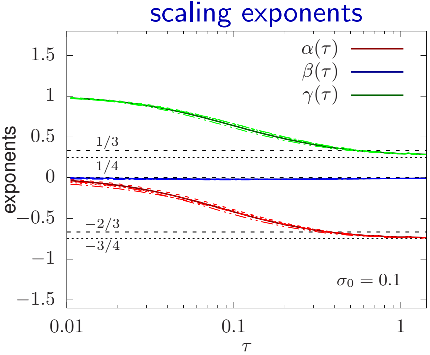
Pre-scaling regime in QCD kinetic theory

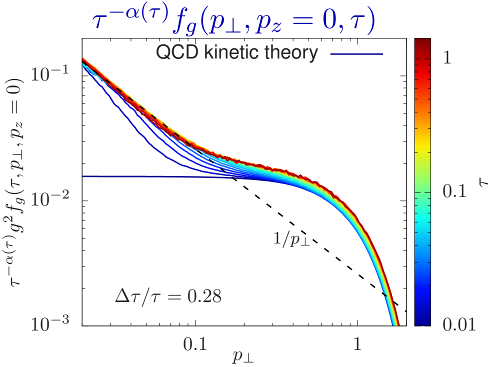
Non-equilibrium dynamics undone by self-similar renormalization

$$f_g(p_\perp, p_\perp, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$

AM and Berges (2019) [13]

Scaling exponents $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$ can be time dependent!





Much earlier collapse to scaling solution f_S — pre-scaling regime.

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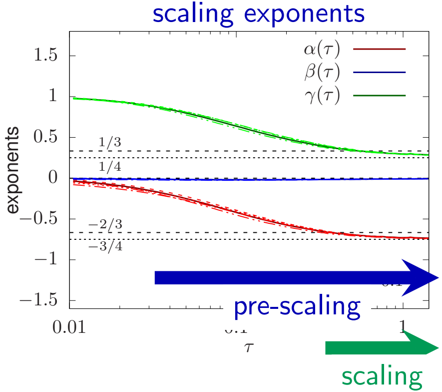
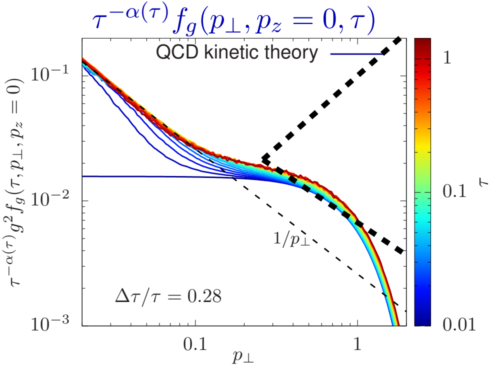
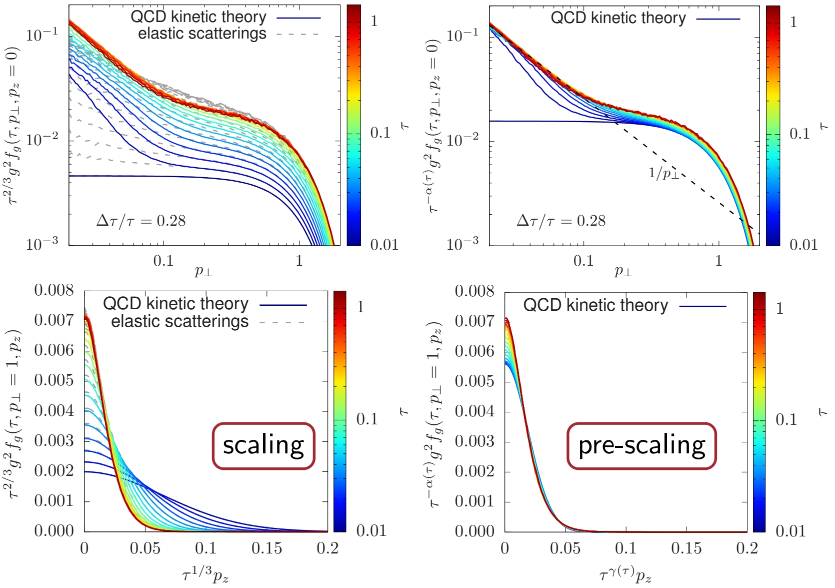




Much earlier collapse to scaling solution f_S — pre-scaling regime.

Comparison between constant and time dependent exponents



Extracting exponents from integral moments

Pre-scaling evolution imposes relations between integral moments

$$n_{m,n}(\tau) \equiv \nu_g \int \frac{d^3\mathbf{p}}{(2\pi)^3}\, p_\perp^m |p_z|^n f_g(p_\perp, p_z, \tau),$$

If $f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$ then

$$\frac{\partial \log n_{m,n}(\tau)}{\partial \log \tau} = \alpha(\tau) - (m+2)\,\beta(\tau) - (n+1)\,\gamma(\tau).$$

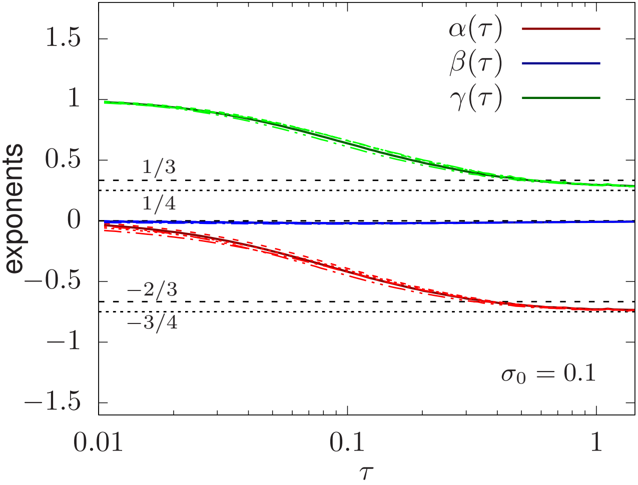
where we redefined the exponents to be $\tau^{\alpha(\tau)} \to \exp\left[\int_1^\tau \frac{d\tau}{\tau}\alpha(\tau)\right]$

If all moments $n_{m,n}$ scale with the same $\alpha, \beta, \gamma \Rightarrow$ pre-scaling regime.

Consider 5 triples of moments: $\{1, p_\perp, |p_z|\}$, $\{1, p_\perp^2, p_z^2\}$, $\{p_\perp, p_\perp^2, p_\perp|p_z|\}$, $\{p_\perp^2, p_\perp^3, p_\perp|p_z|\}$, $\{1, p_\perp^3, |p_z|^3\}$

Time dependent exponents

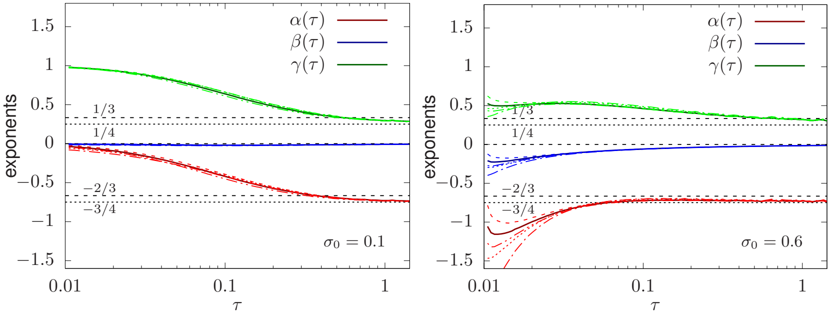
$$f_g(p_\perp, p_\perp, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$



Closely related evolution of moments $n_{m,n}$ with $0 \le n, m \le 3$

Dependence on initial conditions

Vary initial gluon occupation $\sigma_0 = 0.1, 0.6$: $f_g = \frac{\sigma_0}{g^2} e^{-(p_\perp^2 + \xi^2 p_z^2)}$



Time evolution of exponents $\Longrightarrow$ far-from-equilibrium hydrodynamics

$$\partial_\mu T^{\mu\nu}(e, u^\sigma) = 0 \quad \Longleftrightarrow \quad \partial_\mu T^{\mu\nu}(\alpha(\tau), \beta(\tau), \gamma(\tau)) = 0$$

Hydrodynamics, which is not based on expansion around equilibrium!

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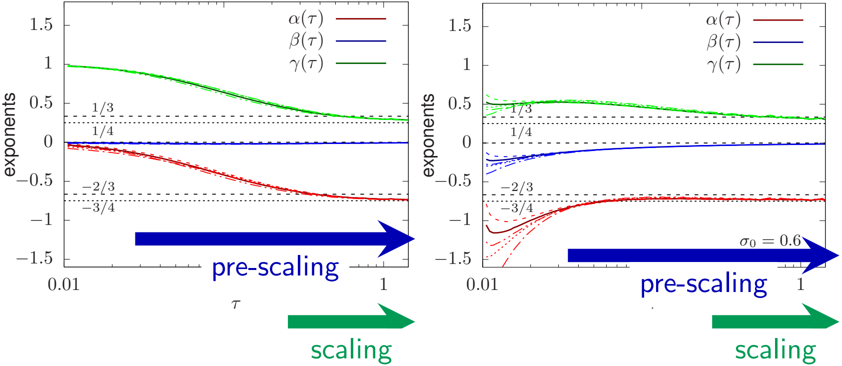

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Hydrodynamics far-from-equilibrium

Integrals of Boltzmann equation $\Rightarrow$ equations of motion for moments

$$\partial_\tau f - \frac{p_z}{\tau}\partial_{p_z} f = -C[f]$$

Consider $J^\mu = \nu_g \int_{\mathbf{p}} \frac{p^\mu}{p^0} f_{\mathbf{p}}, \quad I^{\mu\nu\sigma} = \nu_g \int_{\mathbf{p}} \frac{p^\mu p^\nu p^\sigma}{p^0} f_{\mathbf{p}},$

$$\partial_\tau n + \frac{n}{\tau} = -C_J,$$
$$\partial_\tau I^{\tau xx} + \frac{I^{\tau xx}}{\tau} = -C_I^{xx},$$
$$\partial_\tau I^{\tau zz} + \frac{3I^{\tau zz}}{\tau} = -C_I^{zz},$$

If $f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$ then

$$2\alpha(\tau) + 2\log\tau \frac{\partial\alpha(\tau)}{\partial\log\tau} = -5\frac{\tau C_J}{n} + 2\frac{\tau C_I^{xx}}{I^{\tau xx}} + \frac{\tau C_I^{zz}}{I^{\tau zz}}$$

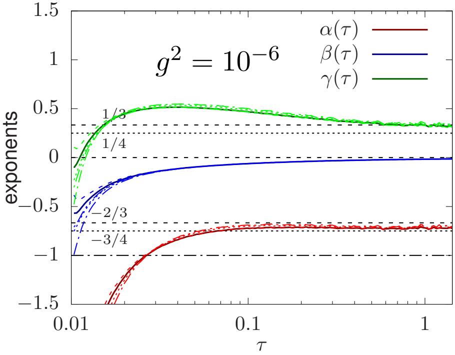
Scaling of the collision kernel closes the system.

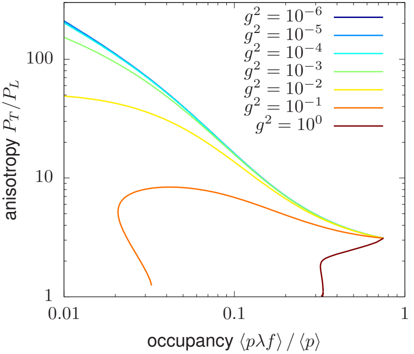
Berges, Mikheev and Mazeliauskas, *work in progress*

Beyond the first stage of Bottom-up

Dependence on the coupling strength (pure glue simulation)

Vary the coupling constant $\alpha_s = g^2/(4\pi)$





"Bottom-up" thermalization scenario

Baier, Mueller, Schiff, and Son (2001)[10]

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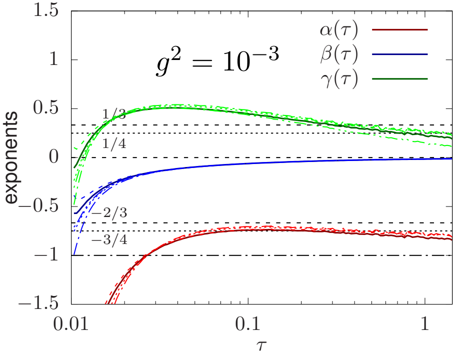

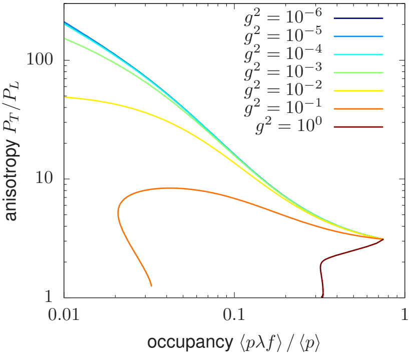

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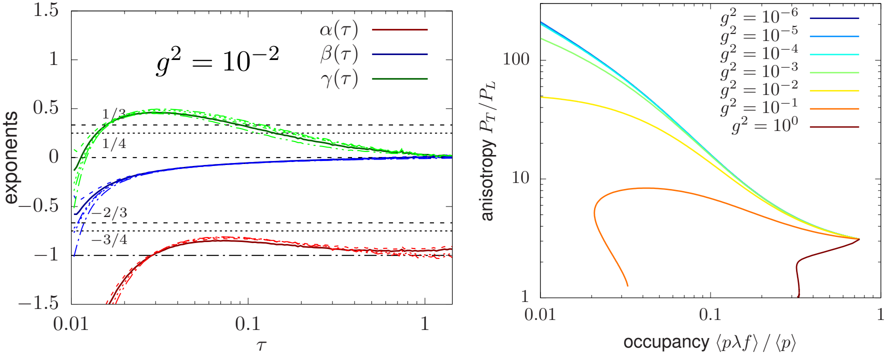

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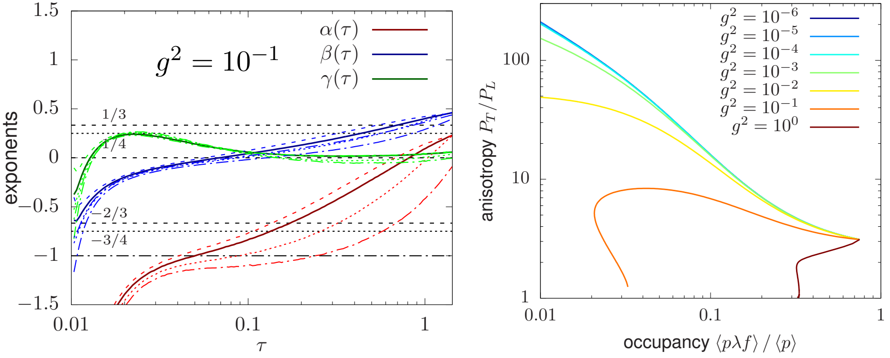

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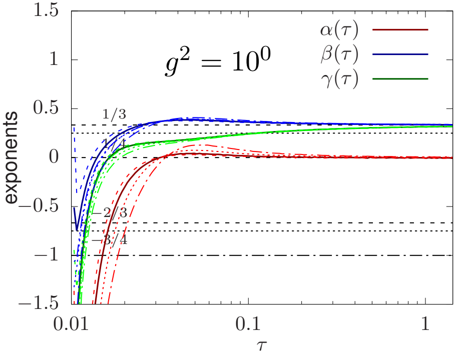

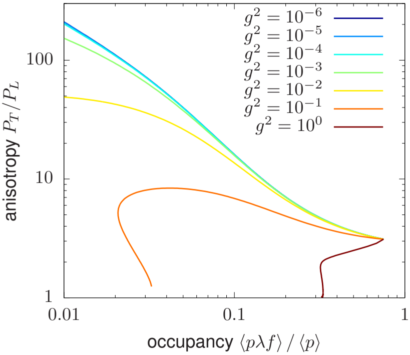

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Outlook on early time dynamics

$$f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z)$$

- Scaling is present in full QCD kinetic theory evolution.
- Found pre-scaling regime — even earlier simplification of non-equilibrium QGP evolution. AM and Berges (2019)

  Pre-scaling in non-relativistic scalars/cold atoms?
- $\alpha(\tau)$, $\beta(\tau)$, $\gamma(\tau)$—new hydrodynamic-like degrees of freedom.

$$\partial_\mu T^{\mu\nu}(e, u^\sigma) = 0 \quad \Longleftrightarrow \quad \partial_\mu T^{\mu\nu}(\alpha, \beta, \gamma) = 0$$

Far-from-equilibrium hydrodynamics?

Berges, Mikheev and Mazeliauskas, *work in progress*

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