k_T-dependent factorization from an amplitude perspective

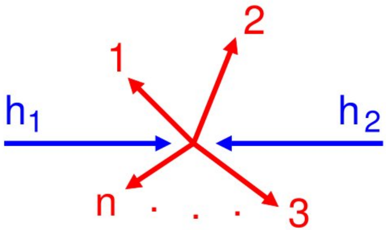
Andreas van Hameren
Institute of Nuclear Physics
Polish Academy of Sciences
Kraków

08-12-2017, Kraków

This work was supported by grant of National Science Center, Poland, No. 2015/17/B/ST2/01838.

Collinear factorization

To separate a perturbatively calculable from the universal in hadron scattering.



PDFs are related to the structure of the hadrons, universal to the scattering process

$$\sigma_{h_1,h_2\to n}(p_1,p_2) = \sum_{a,b}\int dx_1\,dx_2\; f_a(x_1,\mu)\, f_b(x_2,\mu)\; \hat{\sigma}_{a,b\to n}(x_1p_1,x_2p_2;\mu)$$

$$\hat{\sigma}_{a,b\to n}(p_a,p_b;\mu) = \int d\Phi(p_a,p_b\to\{p\}_n)\; |\mathcal{M}_{a,b\to n}(p_a,p_b\to\{p\}_n;\mu)|^2\; \mathcal{O}(p_a,p_b,\{p\}_n)$$

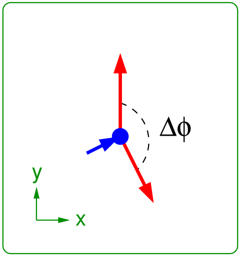
Phase space (includes spin/color summation) governs the kinematics

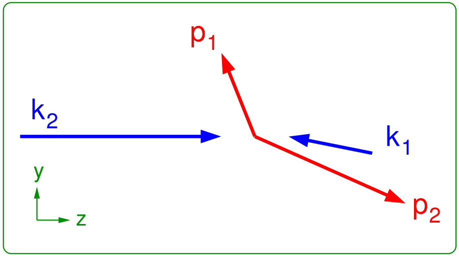
Matrix element (squared) contains model parameters, governs the dynamics

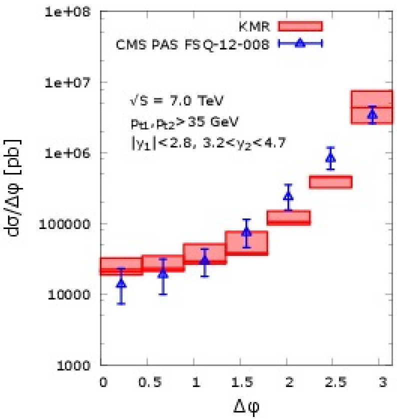
Observable, imposes phase space cuts

Forward-central dijet decorrelations pp $\to$ 2j

AvH, Kutak, Kotko, Sapeta 2014

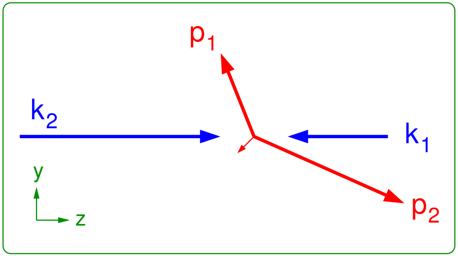


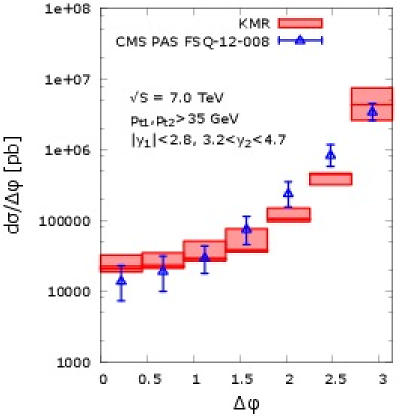




Forward-central dijet decorrelations pp $\longrightarrow$ 2j

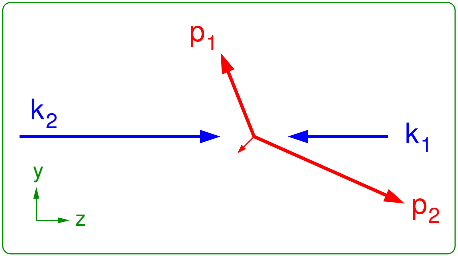
Collinear factorization

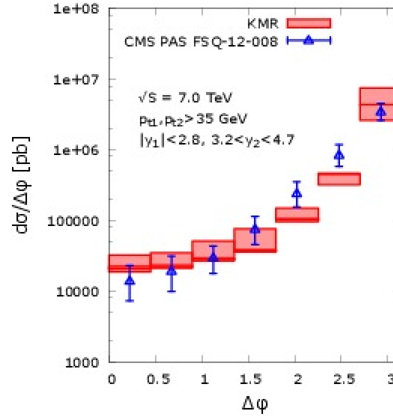




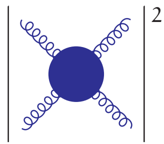
Forward-central dijet decorrelations $pp \to 2j$

Collinear factorization

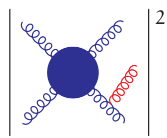




LO: $\hat{\sigma}^{\mathrm{LO}}_{a,b\to n} = \int d\Phi_n \left|\mathcal{M}^{(0)}_{a,b\to n}\right|^2 \mathcal{O}^{\mathrm{LO}}_n$

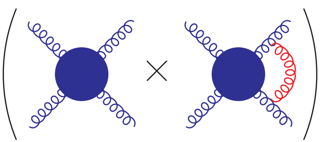


NLO: $\hat{\sigma}^{\mathrm{NLO}}_{a,b\to n} = \int d\Phi_{n+1} \left|\mathcal{M}^{(0)}_{a,b\to n+1}\right|^2 \mathcal{O}^{\mathrm{NLO}}_{n+1}$



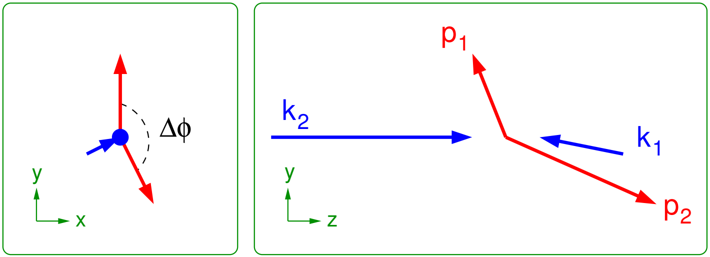
$+ \int d\Phi_n \, 2\mathrm{Re}\left(\mathcal{M}^{(0)}_{a,b\to n}\,\mathcal{M}^{(1)}_{a,b\to n}\right) \mathcal{O}^{\mathrm{LO}}_n$

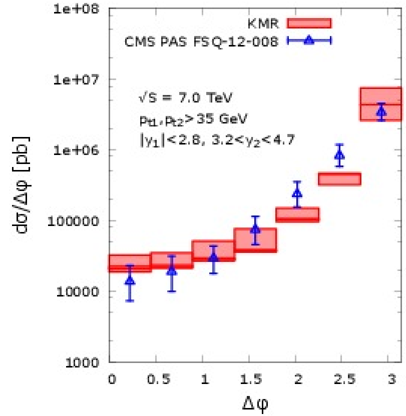
2Re



Forward-central dijet decorrelations $pp \to 2j$

AvH, Kutak, Kotko, Sapeta 2014





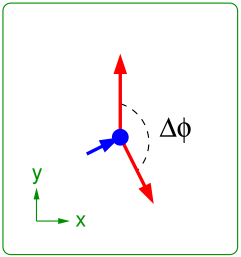
Hybrid factorization:

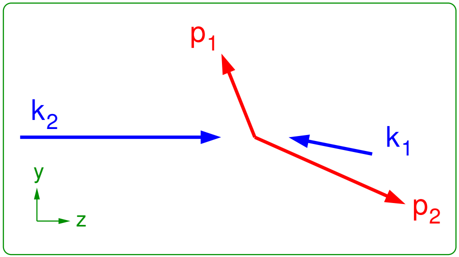
$$d\sigma_{pp\to X} = \int dk_T^2 \int dx_A \int dx_B \sum_b \mathcal{F}_{g^*}(x_A, k_T, \mu)\, f_b(x_B, \mu)\, d\hat{\sigma}_{g^*b\to X}(x_A, x_B, k_T, \mu)$$

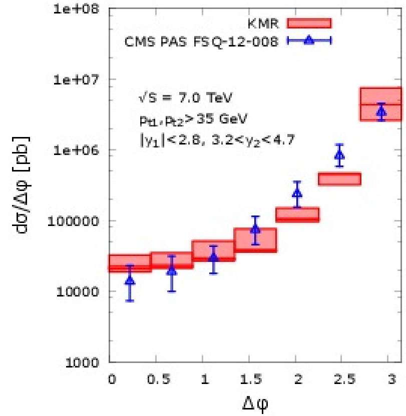
$$k_1^\mu = x_A P_A^\mu + k_T^\mu \qquad P_A^2 = 0 \qquad k_1^2 = k_T^2$$
$$k_2^\mu = x_B P_B^\mu \qquad P_B^2 = 0 \qquad k_2^2 = 0$$

Forward-central dijet decorrelations $pp \to 2j$

AvH, Kutak, Kotko, Sapeta 2014







Hybrid factorization:

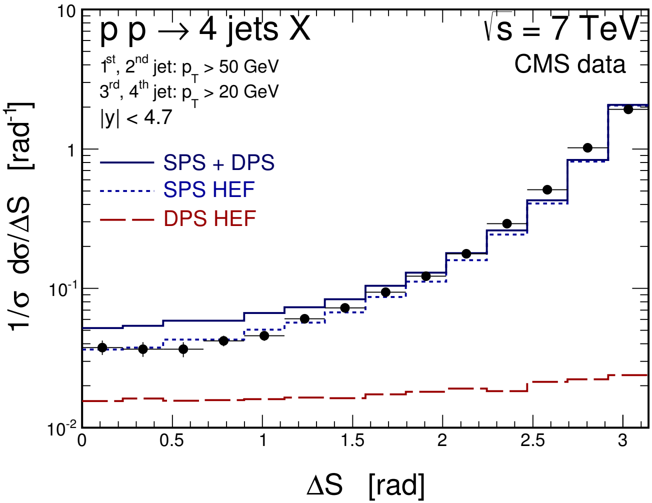
$$d\sigma_{pp\to X} = \int dk_T^2 \int dx_A \int dx_B \sum_b \mathcal{F}_{g^*}(x_A, k_T, \mu)\, f_b(x_B, \mu)\, d\hat{\sigma}_{g^* b\to X}(x_A, x_B, k_T, \mu)$$

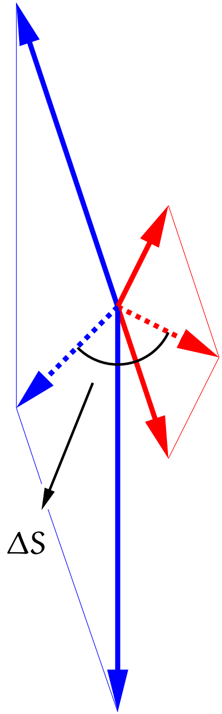
$$k_1^\mu = x_A P_A^\mu + k_T^\mu \qquad P_A^2 = 0 \qquad k_1^2 = k_T^2$$
$$k_2^\mu = x_B P_B^\mu \qquad P_B^2 = 0 \qquad k_2^2 = 0$$

$$x_B \gg x_A \qquad \left|\vec{p}_1 + \vec{p}_2\right| = \left|\vec{k}_T\right|$$

Four jets with k_T-factorization

Maciuła, Szczurek, Kutak, Serino, AvH 2016





- ΔS is the azimutal angle between the sum of the two hardest jets and the sum of the two softest jets.
- This variable has no distribution at LO in collinear factorization: pairs would have to be back-to-back.
- k_T-factorization allows for the necessary momentum inbalance.

Factorization for hadron scattering

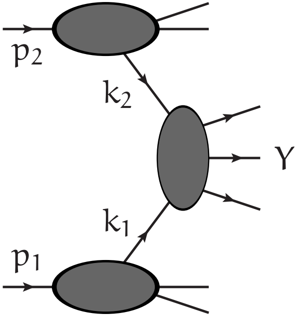
General formula for cross section with $\pi^* \in \{g^*, q^*, \bar{q}^*\}$:

$$d\sigma\big(h_1(p_1)h_2(p_2) \to Y\big) = \sum_{a,b} \int d^4k_1\, \mathcal{P}_{1,a}(k_1) \int d^4k_2\, \mathcal{P}_{2,b}(k_2)\, d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$$

Collinear factorization: $\mathcal{P}_{i,a}(k) = \int_0^1 \frac{dx}{x} f_{i,a}(x,\mu)\, \delta^4(k - x\,p_i)$

k_T-factorization: $\mathcal{P}_{i,a}(k) = \int \frac{d^2\mathbf{k}_T}{\pi} \int_0^1 \frac{dx}{x} \mathcal{F}_{i,a}(x, |\mathbf{k}_T|, \mu)\, \delta^4(k - x\,p_i - k_T)$

- The *parton level* cross section $d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$ can be calculated within perturbative QCD.
- The *parton distribution functions* $f_{i,a}$ and $\mathcal{F}_{i,a}$ must be modelled and fit against data.
- Unphysical scale μ is a price to pay, but its dependence is calculable within perturbative QCD via *evolution equations*.



Factorization for hadron scattering

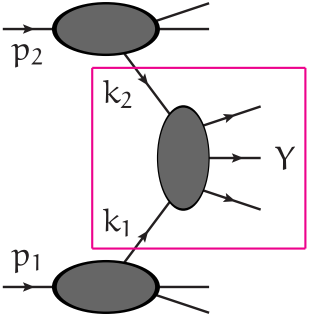
General formula for cross section with $\pi^* \in \{g^*, q^*, \bar{q}^*\}$:

$$d\sigma\big(h_1(p_1)h_2(p_2) \to Y\big) = \sum_{a,b} \int d^4k_1\, \mathcal{P}_{1,a}(k_1) \int d^4k_2\, \mathcal{P}_{2,b}(k_2)\, d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$$

Collinear factorization: $\mathcal{P}_{i,a}(k) = \int_0^1 \frac{dx}{x} f_{i,a}(x,\mu)\, \delta^4(k - x\,p_i)$

k_T-factorization: $\mathcal{P}_{i,a}(k) = \int \frac{d^2\mathbf{k}_T}{\pi} \int_0^1 \frac{dx}{x} \mathcal{F}_{i,a}(x, |\mathbf{k}_T|, \mu)\, \delta^4(k - x\,p_i - k_T)$

$$\hat{\sigma} = \int d\Phi(1,2 \to 3,4,\dots,n)\, \big|\mathcal{M}(1,2,\dots,n)\big|^2\, \mathcal{O}(p_3,p_4,\dots,p_n)$$

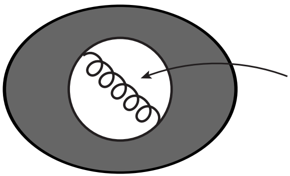


phase space includes summation over color and spin
squared amplitude calculated perturbatively
observable includes phase space cuts, or jet algorithm

Gauge invariance

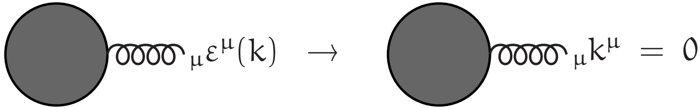
In order to be physically relevant, any scattering amplitude following the constructive definition given before must satisfy the following

Freedom in choice of gluon propagator:



$$\left\{\begin{array}{l} \dfrac{-i}{k^2}\left[g^{\mu\nu} - (1-\xi)\dfrac{k^\mu k^\nu}{k^2}\right] \\[2ex] \dfrac{-i}{k^2}\left[g^{\mu\nu} - \dfrac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n} + (n^2 + \xi k^2)\dfrac{k^\mu k^\nu}{(k\cdot n)^2}\right] \end{array}\right.$$

Ward identity:



- Only holds if all external particles are on-shell.
- k_T-factorization requires off-shell initial-state momenta $k^\mu = p^\mu + k_T^\mu$.
- How to define amplitudes with off-shell intial-state momenta?

Weyl spinors for light-like momenta

Weyl spinors for light-like momenta

$$|p] = \begin{pmatrix} L(p) \\ \mathbf{0} \end{pmatrix} \qquad L(p) = \frac{1}{\sqrt{|p_0 + p_3|}} \begin{pmatrix} -p_1 + ip_2 \\ p_0 + p_3 \end{pmatrix}$$

$$|p\rangle = \begin{pmatrix} \mathbf{0} \\ R(p) \end{pmatrix} \qquad R(p) = \frac{\sqrt{|p_0 + p_3|}}{p_0 + p_3} \begin{pmatrix} p_0 + p_3 \\ p_1 + ip_2 \end{pmatrix}$$

Dual spinors are defined without complex conjugation

$$[p| = \left((\mathcal{E}L(p))^T, \mathbf{0} \right) \qquad \langle p| = \left(\mathbf{0}, (\mathcal{E}^T R(p))^T \right) \qquad \mathcal{E} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$|p\rangle[p| + |p]\langle p| = \not{p} = \gamma_\mu p^\mu$$
$$\langle p||q] = [p||q\rangle = 0$$
$$\langle p||p\rangle = [p||p] = 0$$
$$\not{p}|p\rangle = \not{p}|p] = 0 \quad , \quad \langle p|\not{p} = [p|\not{p} = 0$$
$$p^\mu = \tfrac{1}{2}\langle p|\gamma^\mu|p]$$

$$\langle pq\rangle \equiv \langle p||q\rangle \quad , \quad [pq] \equiv [p||q]$$
$$\langle qp\rangle = -\langle pq\rangle \quad , \quad [qp] = -[pq]$$
$$\langle pq\rangle[qp] = 2p\cdot q$$
$$\langle p|\not{k}|q] = [q|\not{k}|p\rangle$$
$$\langle p|\not{r}|q] = \langle pr\rangle[rq]$$

Schouten identity

$$\frac{|q\rangle\langle p|}{\langle pq\rangle} + \frac{|p\rangle\langle q|}{\langle qp\rangle} + \frac{|q][p|}{[pq]} + \frac{|p][q|}{[qp]} = 1$$

BCFW recursion for on-shell amplitudes

Multi-gluon amplitudes have much simpler expressions than one would expect from the Feynman graphs, in particular the MHV amplitudes:

$$\mathcal{A}(\mathrm{i}^-, \mathrm{j}^-, (\text{the rest})^+) =$$

BCFW recursion for on-shell amplitudes

Multi-gluon amplitudes have much simpler expressions than one would expect from the Feynman graphs, in particular the MHV amplitudes:

$$\mathcal{A}(i^-, j^-, (\text{the rest})^+) = \frac{\langle p_i p_j \rangle^4}{\langle p_1 p_2 \rangle \langle p_2 p_3 \rangle \cdots \langle p_{n-2} p_{n-1} \rangle \langle p_{n-1} p_n \rangle \langle p_n p_1 \rangle}$$

BCFW recursion for on-shell amplitudes

Britto, Cachazo, Feng, Witten 2005

Multi-gluon amplitudes have much simpler expressions than one would expect from the Feynman graphs, in particular the MHV amplitudes:

$$\mathcal{A}(i^-, j^-, (\text{the rest})^+) = \frac{\langle p_i p_j \rangle^4}{\langle p_1 p_2 \rangle \langle p_2 p_3 \rangle \cdots \langle p_{n-2} p_{n-1} \rangle \langle p_{n-1} p_n \rangle \langle p_n p_1 \rangle}$$

BCFW recursion allows for easy construction of such simple expressions

- it is a recursion of *on-shell amplitudes*, rather than off-shell Green functions
- it is most efficiently applied as a recursion of *expressions*
- it is easily proven using Cauchy's theorem

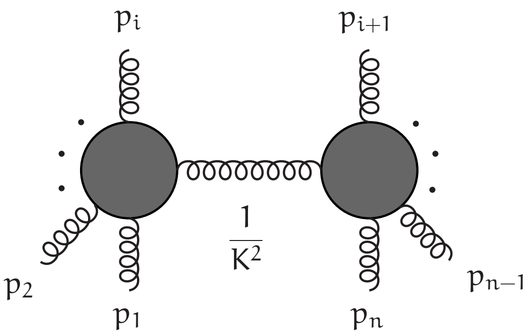
For a rational function f of a complex variable z which vanishes at infinity, we have

$$\oint_R \frac{dz}{2\pi i} \frac{f(z)}{z} \overset{R\to\infty}{=} 0 \qquad \Rightarrow \qquad f(0) = \sum_i \frac{\text{Residue}(f \ @\ z = z_i)}{-z_i}$$

This is applied to amplitudes by turning them into functions of a complex variable by analytical continuation of the momenta to complex values.

BCFW recursion for on-shell amplitudes

Amplitudes have poles at kinematical channels, and the residues factorize into amplitudes.

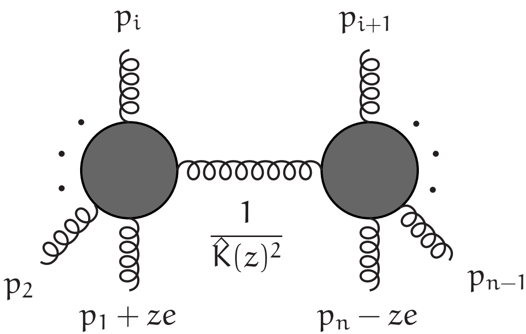


$$K^\mu = p_1^\mu + p_2^\mu + \cdots + p_i^\mu$$
$$= -p_{i+1}^\mu - \cdots - p_{n-1}^\mu - p_n^\mu$$

BCFW recursion for on-shell amplitudes

Britto, Cachazo, Feng, Witten 2005

Amplitudes have poles at kinematical channels, and the residues factorize into amplitudes.



$$\hat{K}^\mu(z) = p_1^\mu + p_2^\mu + \cdots + p_i^\mu + ze^\mu$$
$$= -p_{i+1}^\mu - \cdots - p_{n-1}^\mu - p_n^\mu + ze^\mu$$

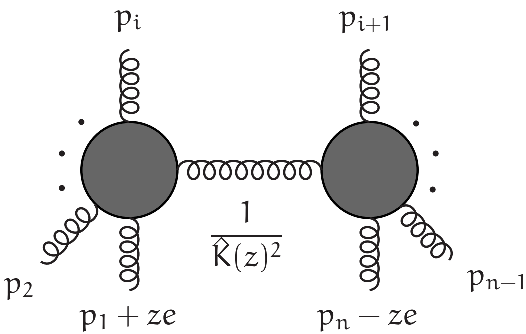
$$e^\mu = \tfrac{1}{2}\langle p_1|\gamma^\mu|p_n]$$

$$\hat{K}(z)^2 = 0 \quad \Leftrightarrow \quad z = -\frac{(p_1 + \cdots + p_i)^2}{2(p_2 + \cdots + p_i)\cdot e}$$

BCFW recursion for on-shell amplitudes

Britto, Cachazo, Feng, Witten 2005

Amplitudes have poles at kinematical channels, and the residues factorize into amplitudes.



$$\hat{K}^\mu(z) = p_1^\mu + p_2^\mu + \cdots + p_i^\mu + ze^\mu$$
$$= -p_{i+1}^\mu - \cdots - p_{n-1}^\mu - p_n^\mu + ze^\mu$$

$$e^\mu = \tfrac{1}{2}\langle p_1|\gamma^\mu|p_n]$$

$$\hat{K}(z)^2 = 0 \quad \Leftrightarrow \quad z = -\frac{(p_1 + \cdots + p_i)^2}{2(p_2 + \cdots + p_i)\cdot e}$$

$$\mathcal{A}(1^+, 2, \ldots, n-1, n^-) = \sum_{i=2}^{n-1} \sum_{h=+,-} \mathcal{A}(\hat{1}^+, 2, \ldots, i, -\hat{K}_{1,i}^h)\, \frac{1}{K_{1,i}^2}\, \mathcal{A}(\hat{K}_{1,i}^{-h}, i+1, \ldots, n-1, \hat{n}^-)$$

$$\mathcal{A}(1^+, 2^-, 3^-) = \frac{\langle 23\rangle^3}{\langle 31\rangle\langle 12\rangle} \quad , \quad \mathcal{A}(1^-, 2^+, 3^+) = \frac{[32]^3}{[21][13]}$$

Amplitudes with off-shell gluons

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$k_1^\mu + k_2^\mu + \cdots + k_n^\mu = 0 \qquad \text{momentum conservation}$$

$$p_1^2 = p_2^2 = \cdots = p_n^2 = 0 \qquad \text{light-likeness}$$

$$p_1 \cdot k_1 = p_2 \cdot k_2 = \cdots = p_n \cdot k_n = 0 \qquad \text{eikonal condition}$$

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$k_1^\mu + k_2^\mu + \cdots + k_n^\mu = 0 \qquad \text{momentum conservation}$$

$$p_1^2 = p_2^2 = \cdots = p_n^2 = 0 \qquad \text{light-likeness}$$

$$p_1\cdot k_1 = p_2\cdot k_2 = \cdots = p_n\cdot k_n = 0 \qquad \text{eikonal condition}$$

With the help of an auxiliary four-vector q^μ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q)p^\mu \quad \text{with} \quad x(q) \equiv \frac{q\cdot k}{q\cdot p}$$

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$\begin{aligned} k_1^\mu + k_2^\mu + \cdots + k_n^\mu &= 0 && \text{momentum conservation} \\ p_1^2 = p_2^2 = \cdots = p_n^2 &= 0 && \text{light-likeness} \\ p_1 \cdot k_1 = p_2 \cdot k_2 = \cdots = p_n \cdot k_n &= 0 && \text{eikonal condition} \end{aligned}$$

With the help of an auxiliary four-vector q^μ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q) p^\mu \quad \text{with} \quad x(q) \equiv \frac{q \cdot k}{q \cdot p}$$

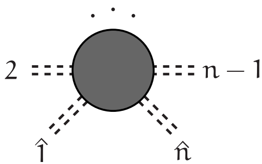
Construct k_T^μ explicitly in terms of p^μ and q^μ:

$$k_T^\mu(q) = -\frac{\kappa}{2}\,\varepsilon^\mu - \frac{\kappa^*}{2}\,\varepsilon^{*\mu} \quad \text{with} \quad \begin{cases} \varepsilon^\mu = \dfrac{\langle p|\gamma^\mu|q]}{[pq]} \quad , \quad \kappa = \dfrac{\langle q|\not{k}|p]}{\langle qp\rangle} \\ \varepsilon^{*\mu} = \dfrac{\langle q|\gamma^\mu|p]}{\langle qp\rangle} \quad , \quad \kappa^* = \dfrac{\langle p|\not{k}|q]}{[pq]} \end{cases}$$

$k^2 = -\kappa\kappa^*$ is independent of q^μ, but also individually κ and κ^* are independent of q^μ.

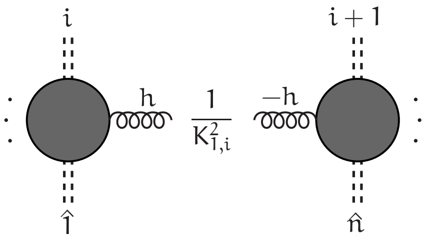
BCFW recursion for off-shell amplitudes

The BCFW recursion formula becomes



$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h}$$

$A_{i,h} =$

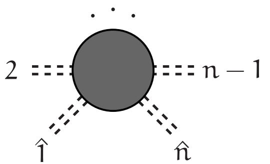


"On-shell condition" for "off-shell" gluons: $p_i \cdot k_i = 0$

BCFW recursion for off-shell amplitudes

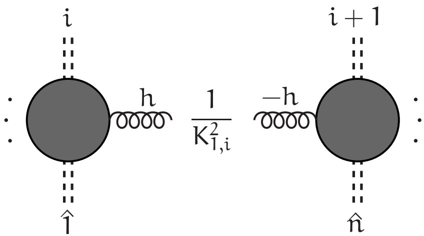
AvH 2014
AvH, Serino 2015

The BCFW recursion formula becomes

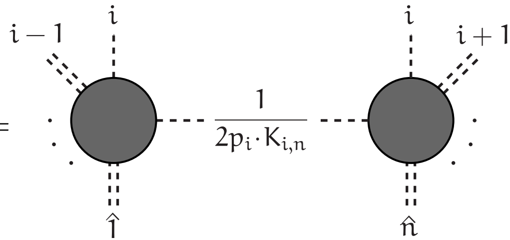


$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i$$

$A_{i,h} =$



$B_i =$

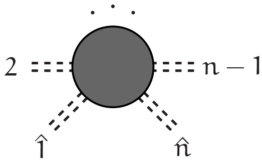


“On-shell condition” for “off-shell” gluons: $p_i \cdot k_i = 0$

BCFW recursion for off-shell amplitudes

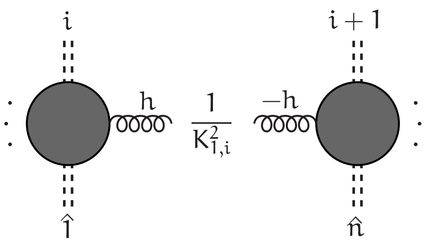
AvH 2014
AvH, Serino 2015

The BCFW recursion formula becomes

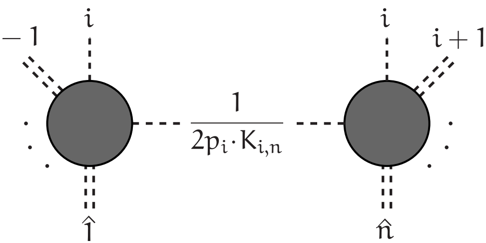


$$= \sum_{i=2}^{n-2}\sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D\,,$$

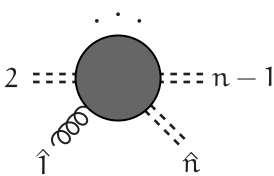
$A_{i,h} =$



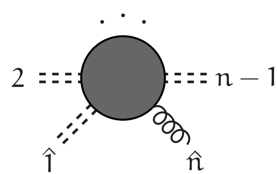
$B_i =$



$C = \dfrac{1}{\kappa_1}$



$D = \dfrac{1}{\kappa_1^*}$



Example of a 4-gluon amplitude

$$\mathcal{A}(1^*, 2^-, 3^*, 4^+) =$$

Example of a 4-gluon amplitude

$$\mathcal{A}(1^*, 2^-, 3^*, 4^+) = \frac{\langle 13\rangle^3 [13]^3}{\langle 34\rangle\langle 41\rangle\langle 1|\not{k}_3 + \not{p}_4|3]\langle 3|\not{k}_1 + \not{p}_4|1][32][21]}$$
$$+ \frac{1}{\kappa_1^* \kappa_3} \frac{\langle 12\rangle^3 [43]^3}{\langle 2|\not{k}_3|4]\langle 1|\not{k}_3 + \not{p}_4|3](k_3 + p_4)^2} + \frac{1}{\kappa_1 \kappa_3^*} \frac{\langle 23\rangle^3 [14]^3}{\langle 2|\not{k}_1|4]\langle 3|\not{k}_1 + \not{p}_4|1](k_1 + p_4)^2}$$

- Eventual matrix element needs factor $k_1^2 k_3^2 = |\kappa_1|^2 |\kappa_3|^2$.
  This *must not* be included at the amplitude level not to spoil analytic structure.

- Last two terms dominate for $|k_1| \to 0$ and $|k_3| \to 0$, and give the on-shell helicity amplitudes in that limit.

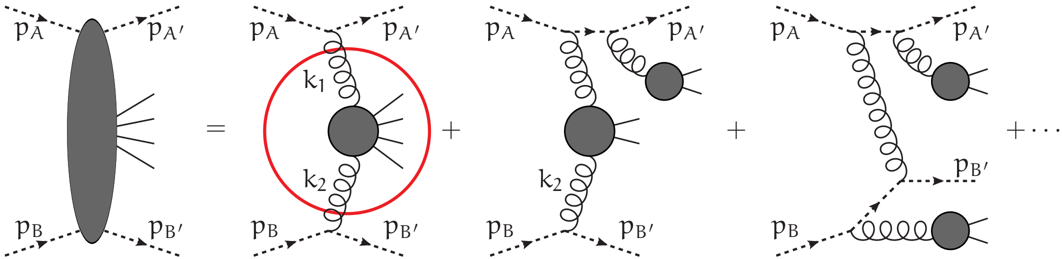
$$\mathcal{A}(1^*, 2^-, 3^*, 4^+) \xrightarrow{|k_1|,|k_3| \to 0} \frac{1}{\kappa_1^* \kappa_3} \mathcal{A}(1^-, 2^-, 3^+, 4^+) + \frac{1}{\kappa_1 \kappa_3^*} \mathcal{A}(1^+, 2^-, 3^-, 4^+)$$

- Coherent sum of amplitudes becomes incoherent sum of squared amplitudes via angular integrations for $\vec{k}_{1T}$ and $\vec{k}_{3T}$.

Amplitude as embedding

AvH, Kutak, Kotko 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



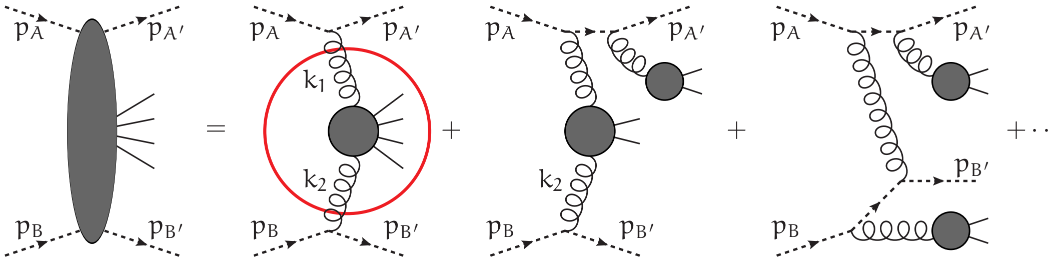
$$p_A^\mu = \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu}$$

$$p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu$$

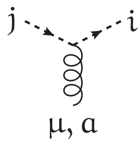
Amplitude as embedding

AvH, Kutak, Kotko 2013

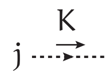
Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



$$\left.\begin{aligned} p_A^\mu &= \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} \\ p_{A'}^\mu &= -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu \\ \Lambda &\to \infty \end{aligned}\right\} \Rightarrow$$



$$= -i\, T^a_{i,j}\, p_1^\mu$$

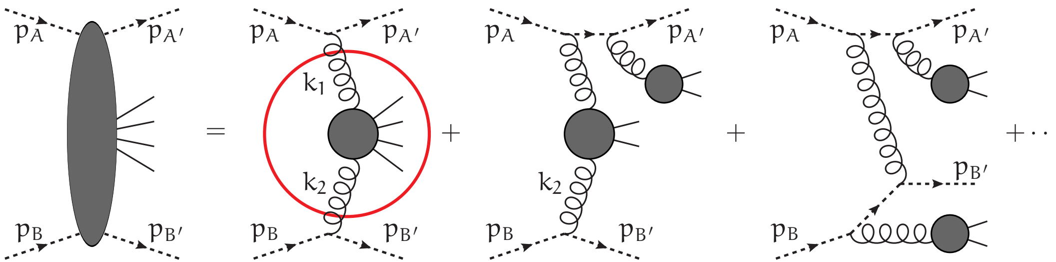


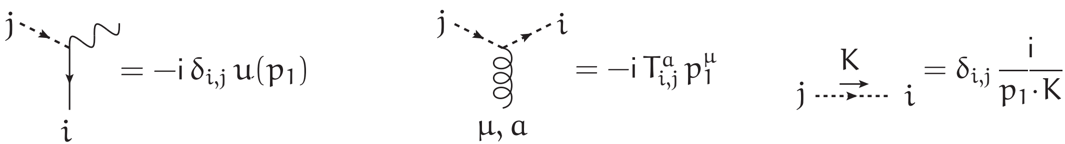
$$= \delta_{i,j}\,\frac{i}{p_1\cdot K}$$

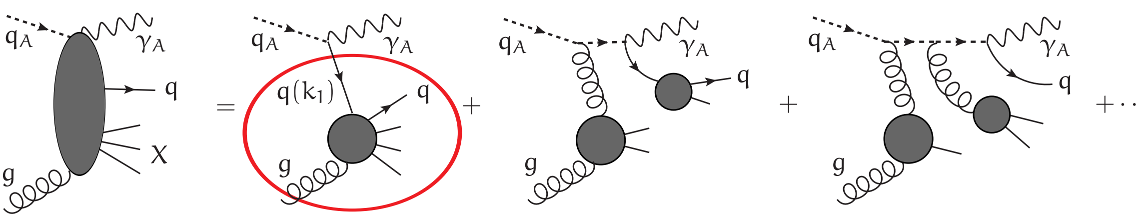
Amplitude as embedding

AvH, Kutak, Kotko 2013
AvH, Kutak, Salwa 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



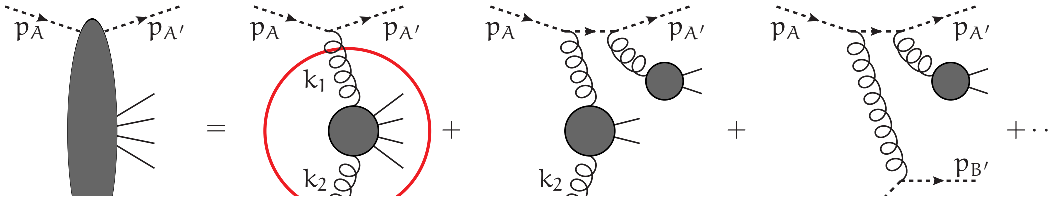




Amplitude as embedding

AvH, Kutak, Kotko 2013
AvH, Kutak, Salwa 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

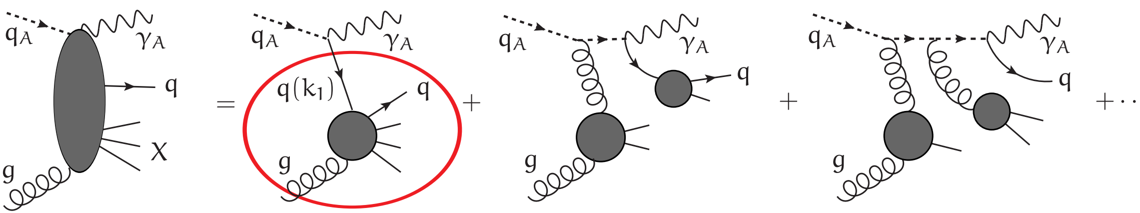


p_B

j

In agreement with the effective action approach of
Lipatov 1995, Antonov, Lipatov, Kuraev, Cherednikov 2005
Lipatov, Vyazovsky 2000, Nefedov, Saleev, Shipilova 2013
and the Wilson-line approach of
Kotko 2014

$i \;=\delta_{i,j}\,\frac{i}{p_1\cdot K}$



KaTie https://bitbucket.org/hameren/katie

- parton level event generator, like Alpgen, Helac, MadGraph, etc.
- arbitrary processes within the standard model (including effective Hg) with several final-state particles.
- 0, 1, or 2 off-shell intial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids, or with TMDlib Hautmann, Jung, Krämer, Mulders, Nocera, Rogers, Signori 2014.
- a calculation is steered by a single input file.
- employs an optimization stage in which the pre-samplers for all channels are optimized.
- during the generation stage several event files can be created in parallel.
- can generate (naively factorized) MPI events.
- event files can be processed further by parton-shower program like CASCADE.

```
Ngroup = 1
Nfinst = 3
process = g u  -> mu+ mu- u   factor = 1   groups = 1   pNonQCD = 2 0 0
process = g u~ -> mu+ mu- u~  factor = 1   groups = 1   pNonQCD = 2 0 0
process = g d  -> mu+ mu- d   factor = 1   groups = 1   pNonQCD = 2 0 0
process = g d~ -> mu+ mu- d~  factor = 1   groups = 1   pNonQCD = 2 0 0
lhaSet = MSTW2008nlo68cl
offshell = 1 0
tmdTableDir = /home/user0/kTfac/tables/krzysztof02/
tmdpdf = g  KMR_gluon.dat
tmdpdf = u  KMR_u.dat
tmdpdf = u~ KMR_ubar.dat
tmdpdf = d  KMR_d.dat
tmdpdf = d~ KMR_dbar.dat
tmdpdf = s  KMR_s.dat
tmdpdf = s~ KMR_sbar.dat
tmdpdf = c  KMR_c.dat
tmdpdf = c~ KMR_cbar.dat
tmdpdf = b  KMR_b.dat
tmdpdf = b~ KMR_bbar.dat
Nflavors = 5
helicity = sampling
Noptim = 1,000,000
Ecm = 7000
Esoft = 20
```

```
cut = {deltaR|1,3|} > 0.4
cut = {deltaR|2,3|} > 0.4
cut = {pT|1|} > 20
cut = {pT|2|} > 20
cut = {pseudoRap|1|} > 2.0
cut = {pseudoRap|2|} > 2.0
cut = {pseudoRap|1|} < 4.5
cut = {pseudoRap|2|} < 4.5
cut = {mass|1+2|} > 60
cut = {mass|1+2|} < 120
cut = {pT|3|} > 20
cut = {rapidity|3|} > 2.0
cut = {rapidity|3|} < 4.5
scale = ({pT|3|}+{pT|1+2|}+91.1882D0)/3
mass = Z 91.1882 2.4952
mass = W 80.419 2.21
mass = H 125.0 0.00429
mass = t 173.5
switch = withQCD Yes
switch = withQED Yes
switch = withWeak Yes
switch = withHiggs No
switch = withHG No
coupling = Gfermi 1.16639d-5
```

Example steering file:
$p\,p \to \mu^+\mu^-\,j$ in the forward direction

- has been used in several studies
  - *Four-jet production in single- and double-parton scattering within high-energy factorization*, Kutak, Maciula, Serino, Szczurek, AvH 2016
  - *Associated production of D-mesons with jets at the LHC*, Maciula, Szczurek 2017
  - *Towards tomography of quarkgluon plasma using double inclusive forward-central jets in PbPb collision*, Deák, Kutak, Tywoniuk 2017
  - *Single- and double-scattering production of four muons in ultraperipheral PbPb collisions at the Large Hadron Collider*, AvH, Kusek-Gawenda, Szczurek 2017
- covers complete parton-level phase space; no deformation of final-state momenta required when interfacing with initial-state parton shower
  - *Calculations with off-shell matrix elements, TMD parton densities and TMD Parton showers*, Bury, AvH, Jung, Kutak, Sapeta, Serino in preparation
- one can use an arbitrary initial-state parton shower and re-weight events
- can be used in "on-shell" mode, and is then equivalent to, say, tree-level MadGraph

Off-shell one-loop amplitudes

Off-shell one-loop amplitudes

Initial steps have already been taken in the *parton reggeization approach* employing Lipatov's effective action.
Hentschinski, Sabio Vera 2012
Chachamis, Hentschinski, Madrigal, Sabio Vera 2012
Nefedov, Saleev 2017

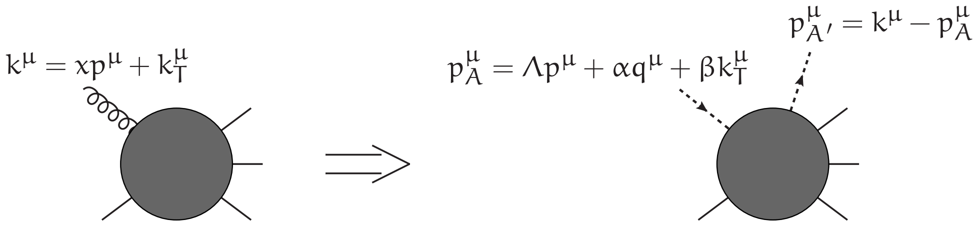
The main problem is caused by linear denominators in loop integrals

$$\int d^{4-2\varepsilon}\ell \, \frac{\cdots}{\cdots \, p\cdot(\ell+K) \, \cdots}$$

and the divergecies they cause. In particular one would like to use a regularization that

- is manifestly Lorentz covariant
- manifestly preserves gauge invariance
- can be used incombination with dimensional regularization
- is practical

Off-shell one-loop amplitudes



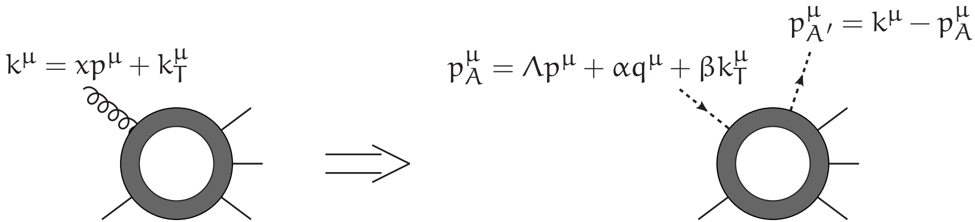
where p, q are light-like with $p\cdot q > 0$, where $p\cdot k_T = q\cdot k_T = 0$, and where

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2} \quad , \quad \beta = \frac{1}{1+\sqrt{1-x/\Lambda}} \quad \Longrightarrow \quad \begin{cases} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = xp^\mu + k_T^\mu \end{cases}$$

for any value of the parameter Λ. Auxiliary quark propagators become eikonal for $\Lambda \to \infty$:

$$i\,\frac{\not{p}_A + \not{K}}{(p_A+K)^2} = \frac{i\,\not{p}}{2p\cdot K} + \mathcal{O}(\Lambda^{-1})$$

Off-shell one-loop amplitudes



where p, q are light-like with $p\cdot q > 0$, where $p\cdot k_T = q\cdot k_T = 0$, and where

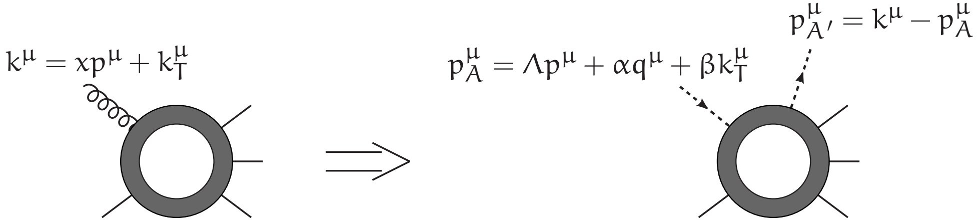
$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2} \quad , \quad \beta = \frac{1}{1+\sqrt{1-x/\Lambda}} \quad \Longrightarrow \quad \begin{cases} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = xp^\mu + k_T^\mu \end{cases}$$

for any value of the parameter Λ. Auxiliary quark propagators become eikonal for $\Lambda \to \infty$:

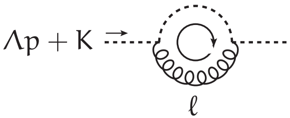
$$\frac{i\not{p}}{2p\cdot(\ell+K)} = i\,\frac{\not{p}_A + \not{\ell} + \not{K}}{(p_A+\ell+K)^2} + \mathcal{O}(\Lambda^{-1})$$

- Λ-parametrization provides natural regularization for linear denominators in loop integrals.
- Taking this limit after loop integration will lead to singularities $\log\Lambda$.

Off-shell one-loop amplitudes



Integrand-based reduction methods cannot be applied with naïve limit $\Lambda \to \infty$ on integrand. For example, the integrand of the following graph (Feynman gauge) vanishes in that limit, but the integral does not:



$$= \int d^{4-2\varepsilon}\ell \frac{\langle p|\gamma^\mu(\not\ell + \Lambda \not p + \not K)\gamma_\mu|p]}{\ell^2(\ell + \Lambda p + K)^2}$$

$$= 2p\cdot K\left[\log\Lambda - \frac{1}{\varepsilon} - 1 + \log\left(-\frac{2p\cdot K}{\mu^2}\right) + \mathcal{O}(\varepsilon)\right]$$

But $\langle p|\gamma^\mu \not p \gamma_\mu|p] = 0$, so naïve power counting in Λ does not work.

Behavior of the scalar integrals

$$\int \frac{d^{4-2\varepsilon}\ell}{\ell^2(\ell+K_1)^2(\ell+\Lambda p+K_2)^2(\ell+\Lambda p+K_3)^2} = \frac{a\log^2\Lambda + b\log\Lambda + c + \mathcal{O}(\Lambda^{-1})}{\Lambda^2}$$

$$\int \frac{d^{4-2\varepsilon}\ell}{\ell^2(\ell+K_1)^2(\ell+K_2)^2(\ell+\Lambda p+K_3)^2} = \frac{a\log^2\Lambda + b\log\Lambda + c + \mathcal{O}(\Lambda^{-1})}{\Lambda}$$

$$\int \frac{d^{4-2\varepsilon}\ell}{\ell^2(\ell+K_1)^2(\ell+\Lambda p+K_2)^2} = \frac{a\log^2\Lambda + b\log\Lambda + c + \mathcal{O}(\Lambda^{-1})}{\Lambda}$$

$$\int \frac{d^{4-2\varepsilon}\ell}{\ell^2(\ell+\Lambda p+K_1)^2} = b\log\Lambda + c + \mathcal{O}(\Lambda^{-1})$$

Conclusions

Tree-level parton-level event generation is the easy part of k_T-dependent factorization.