Towards holography for quantum mechanics
a toy model for AdS/CFT

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Outline

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Motivation

The original AdS/CFT correspondence

Questions on holography

Requirements for a holographic description

Partition function

Correlation functions

The “gravity” subsector

Holographic description for a quantum-mechanical free particle

Conclusions and outlook

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Supersymmetric gauge theory in ordinary 4-dimensional Minkowski spacetime ($\mathcal{N} = 4$ Super Yang-Mills)	$\equiv$	**Superstrings** on a curved 10-dimensional spacetime $AdS_5 \times S^5$

Maldacena '97

- The AdS/CFT correspondence postulates the equivalence of these two theories!
- It is extremely surprising as the two theories are apparently completely different
- In this context string theory does **not** introduce any new **additional** physics but is a dual description of an ordinary (supersymmmetric) gauge theory
- In particular the AdS/CFT correspondence (and its utility) is completely independent whether superstring theory is indeed a description of our physical world (in the sense of a grand unified theory) or not...

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What are the origins of the AdS/CFT correspondence?

The origins of the AdS/CFT correspondence

- In superstring theory there exist 4-dimensional objects – so called **D3 branes** which are characterized by the property that strings can end on them
- Massless excitations of strings attached to a D3-brane correspond exactly to fields in $\mathcal{N} = 4$ Super-Yang-Mills gauge theory
- Apart from these excitations there exist an infinite tower of additional massive fields with masses $m^2 = n/\alpha'$
- Now take the above configuration and take the limit $\alpha' \to 0$
  Then these additional fields will decouple from the dynamics!
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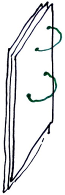

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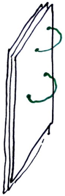

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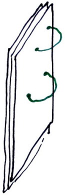

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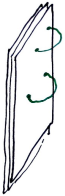

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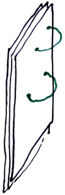

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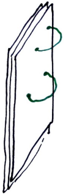

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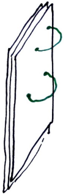

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- One obtains a concrete explicit geometry which is a solution of supergravity Einstein's equations (gravity+appropriate matter fields)
- We take the same limit as before $\alpha' \to 0$
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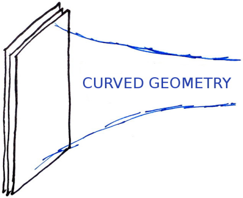

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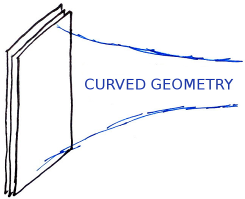

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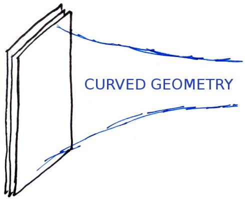

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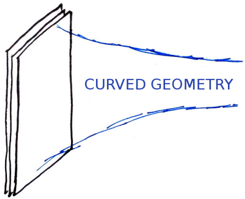

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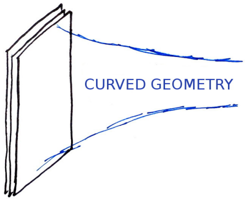

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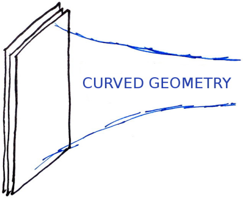

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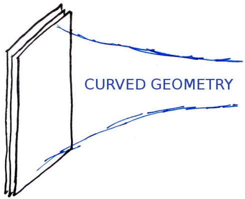

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$\mathcal{N} = 4$ Super Yang-Mills	$\equiv$	Superstrings on $AdS_5 \times S^5$ spacetime
strong coupling nonperturbative regime very difficult weak coupling 'easy'		(semi-)classical strings and/or (super)gravity 'easy' highly quantum regime very difficult

- Provides effective calculational techniques for studying gauge theory dynamics in the nonperturbative regime
- Unexpected close ties between gauge theory and General Relativity
- Apart from any practical utility, the equivalence of two such completely different theories is fascinating theoretically
- We can look at the AdS/CFT correspondence as a highly nontrivial reformulation of gauge theory dynamics in terms of new ('composite') degrees of freedom

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$\mathcal{N} = 4$ Super Yang-Mills $\equiv$ Superstrings on $AdS_5 \times S^5$

- Degrees of freedom on the string side of the correspondence:
  1. massless sector: (super)gravity fields in $AdS_5 \times S^5$
  2. an infinite tower of massive fields (massive closed string excitations)
- In the nonperturbative gauge theory regime of large coupling, these massive fields become very heavy and effectively decouple from the dynamics
- Consequently the dual description of nonperturbative dynamics dramatically simplifies and reduces to just (super)gravity!
- In case of small gauge coupling we cannot neglect the massive string states...

Advantage: We can study the most difficult nonperturbative regime of gauge theory

Disadvantage: It is very difficult to prove the AdS/CFT correspondence... no proof so far...

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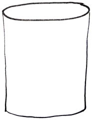

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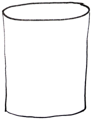

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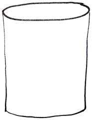

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- The dual description of thermal plasma ($\mathcal{N} = 4$ SYM at nonzero temperature) at large N_c, strong coupling is given by a planar black hole solution
- What is the dual description of thermal plasma still at large N_c but for $\lambda \to 0$?
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There exist potentially simpler versions of holography in lower number of dimensions...

O(N) - higher spin duality

Klebanov, Polyakov

- The singlet sector of free scalar $O(N)$ vector model in 3D – dual to 4D Vasiliev gravity
- Very nontrivial check of 3-point correlation functions   Giombi, Yin
- Very intriguing – first time no strings directly involved
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It would be very interesting to construct a holographic model where the bulk action would be completely known...

Goal:

- Attempt a holographic description for the simplest possible theory that one could think of...

What do we mean by a holographic description?

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Requirements for a holographic description

Suppose that the field theory is defined on some fixed d-dimensional spacetime geometry Σ

I **Equality of partition functions**

- The dual holographic theory should be defined on a higher dimensional manifold M, having Σ as a boundary.
- We should have equality of partition functions

$$Z_{boundary}(\Sigma) = Z_{bulk}(M)$$

- E.g this would provide a bulk interpretation of the thermodynamics of the theory...

Typically for AdS/CFT we want much more...

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- We should be able to compute correlation functions for operators in the boundary theory from the bulk theory

IIb The generating function for correlation functions

Gubser, Klebanov, Polyakov; Witten

- Local observables/operators in the boundary theory should be associated to fields in the bulk theory
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Requirements for a holographic description

III Identification of a gravitational subsector

- The boundary theory is defined on a manifold Σ with fixed metric
- There should be a bulk field associated with the energy-momentum tensor and the boundary metric on Σ
- This would define a gravitational subsector in the bulk theory
- Standard example: Fefferman-Graham expansion of the bulk metric

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2} + \ldots$$

where

$$g_{\mu\nu}(x^\rho, z) = g^{(0)}_{\mu\nu}(x^\rho) + g^{(2)}_{\mu\nu}(x^\rho)\, z^2 + g^{(4)}_{\mu\nu}(x^\rho)\, z^4 + \ldots$$

- For higher spin gravity the whole picture is more complex...
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- The boundary theory is defined on a manifold Σ with fixed metric
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Aim:

Try to satisfy the above requirements **I-III** for one of the simplest systems possible, the quantum mechanical free particle in one dimension.

- Direct (but much simpler) analog of the massless free boson (abelian WZW/CS)
- Extremely simplified system – no spatial direction
- No large N, or coupling — expect the dual description to be quantum

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The system

$$S = \int dt \; \frac{1}{2}\dot{q}^2$$

- Consider the bulk spacetime to be of the form

$$M = \{(t, z) : z \geq 0\}$$

- Since in the 2D massless boson case we have dual abelian Chern-Simons, here we expect to have a 2D abelian BF topological theory

$$S_{BF} = \int_M B \, dA = \int B \, (\partial_t A_z - \partial_z A_t) dtdz$$

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Step I – partition functions

- We will impose the following boundary conditions for the BF theory

$$B = -A_t \mid_{z=0} \qquad\qquad A_t = 0 \mid_{z\to\infty}$$

- Again in analogy to WZW/CS, we have to supplant the BF action with a boundary term so that the variation at the boundary vanishes

$$S^I_{bulk} = S_{BF} + \frac{1}{2}\int_{\{z=0\}} B^2 dt$$

- The Lagrange multiplier field B imposes the constraint $dA = 0$, hence we may set

$$A_z = -\partial_z \Phi \qquad\qquad A_t = -\partial_t \Phi$$

- The boundary values of $\Phi(t,z)|_{z=0}$ will be identified with $q(t)$ hence the partition functions coincide as on the constraint surface

$$S^I_{bulk} = 0 + \frac{1}{2}\int_{\{z=0\}} B^2 dt = \frac{1}{2}\int_{\{z=0\}} A_t^2 dt = \int dt\, \frac{1}{2}\dot{q}^2$$

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Step II – bulk fields for sources

- Consider generating functions of all correlators of $q(t)$

$$\int dt\, \frac{1}{2}\dot{q}^2 + \int dt\, j(t)q(t)$$

- We would like to introduce a new bulk field associated with the source $j(t)$
- In terms of the BF theory gauge field, the particle position $q(t)$ can be understood essentially as a Wilson line

$$\int_{z=0}^{\infty} A_z\, dz = -\int_{z=0}^{\infty} \partial_z \Phi(t,z) = \Phi(t,0) - \Phi(t,\infty) \to \Phi(t,0)$$

- So we have

$$q(t) = \int_L A$$

where the line L is attached to the boundary at time t and goes to infinity in the bulk.

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$$\int dt\, j(t) q(t)$$

we will need two ingredients

- We will introduce another two-dimensional abelian BF theory

$$\int C\, d\alpha$$

as we need a bulk field going over to $j(t)$ at the boundary...

- We use the global 1-form dt (this will be modified later)
- Introduce a constraint term in the action

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- Now the flatness condition $d\alpha = 0$ ensures $\alpha = j(t)dt$, so we can generate the wanted term from a simple bulk interaction between α and A:

$$\int_M \alpha \wedge A = \int_M j(t)dt \wedge (A_t dt + A_z dz) = \int j(t) \int_0^\infty A_z dzdt = \int j(t)q(t)dt$$

- At this stage the overall bulk action is

$$S^{II}_{bulk} = \int_M (B\, dA + C\, d\alpha + \alpha \wedge A + D\, \alpha \wedge dt) + \frac{1}{2} \int_{\partial M} B^2 dt$$

- The appearance of an explicit dt is not very pleasing – but we will get rid of it shortly

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Step III – the "gravity" subsector

- Since the quantum mechanical path integral is essentially just a QFT on a 1-dimensional worldline, one can introduce a fixed 1-dimensional metric $g_{tt}(t)$ and write the action as

$$\frac{1}{2}\int \sqrt{g}\, g^{tt}(\partial_t q)^2 = \frac{1}{2}\int \frac{1}{e}\dot{q}^2$$

  and the einbein $e = e(t)$ is a given function of time...
- We would like to introduce a natural bulk field which goes over to the einbein at the boundary.
- At the same time we will replace the 1-form dt (which is necessarily closed)
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- Since the quantum mechanical path integral is essentially just a QFT on a 1-dimensional worldline, one can introduce a fixed 1-dimensional metric $g_{tt}(t)$ and write the action as

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$$A_t + \eta_t B = 0|_{z=0}$$

and fix the boundary value of η_t

- Accordingly we need to modify the additional boundary action

$$\frac{1}{2}\int_{\{z=0\}} B^2 dt \longrightarrow \frac{1}{2}\int_{\partial M} B^2\,\eta$$

(this works as $\delta\eta_t = 0|_{z=0}$)

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Step IV – integrate out boundary degrees of freedom

- Ultimately we should integrate out B and A to obtain the final bulk action involving only the bulk fields corresponding to sources for $q(t)$ and the energy-momentum tensor T_{tt}

$$e^{iS^{eff}_{bulk}[C,D,E,\alpha,\eta]} = \int DB\,DA\,e^{iS^{III}_{bulk}[B,A,C,D,E,\alpha,\eta]}$$

- Unfortunately this seems to be quite nonlocal...
- One can speculate whether this is a generic situation and a local holographic bulk action in this sense occurs only in special circumstances??? (like large N and/or strong coupling?)

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- We have constructed a dual description of a quantum mechanical free particle which realizes formally some basic requirements for holography
- The bulk fields include a source for the field $q(t)$
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- N components/singlet? relation to 2D Vasiliev
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