Theory and phenomenology of small-x TMD factorization

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in collaboration with:

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Plan

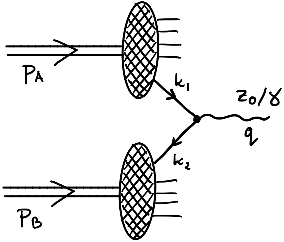
1. Introduction
   The collinear factorization and the Transverse Momentum Dependent (TMD) factorization theorem for Drell-Yan process.
2. Nonuniversality of TMD PDFs
3. TMD factorization at small x
4. Applications for dijets at LHC in pA and UPC

Part I

Introduction

Introduction

The Drell-Yan process



Suppose we want to calculate $d\sigma/dq_T$ spectrum of the Z_0/γ system.

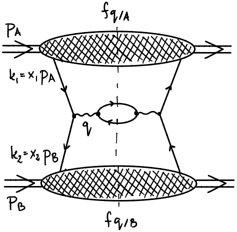
Two different regimes:

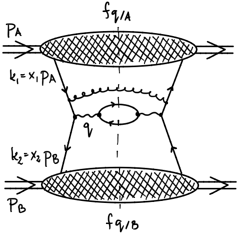
- high q_T (collinear factorization)
- low q_T (TMD factorization)

Introduction

Collinear factorization

$$d\sigma_{AB} \sim \int dx_1 dx_2 \; f_{q/A}\,(x_1,\mu)\, d\hat{\sigma}_{qq}(x_1,x_2,\mu)\, f_{q/B}\,(x_2,\mu)$$



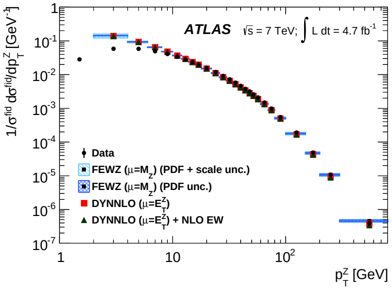


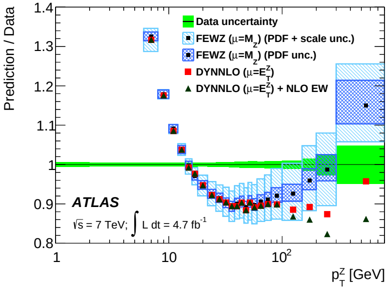
$+ \;\ldots$

The domain: $\mu \sim M \sim q_T \gg \Lambda_{QCD}$

Introduction

Collinear factorization

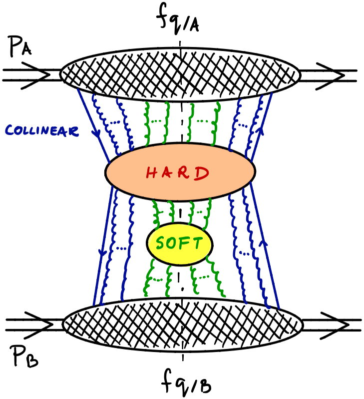




[ATLAS, JHEP09(2014)145]

Introduction

Proving factorization is complicated



Introduction

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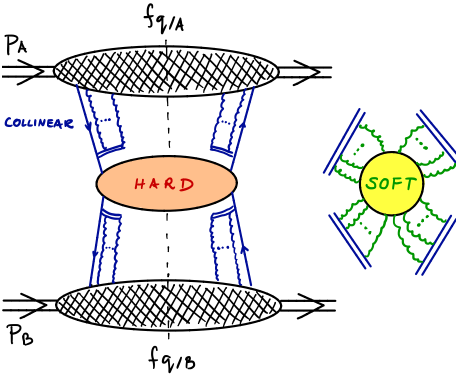

(i) Resummed collinear gluons are included into PDFs,
(ii) soft factor is shown to be equal to 1.

Introduction

Region $q_T \ll \mu \sim M$

Collinear factorization fails – large logs $\log(q_T/\mu)$

- the soft factor S does not vanish
- the PDFs depend on the transverse momenta

$$\frac{d\sigma}{dq_T^2} \sim \int dx_1 dx_2 \; d\hat{\sigma}_{qq} \int d^2 b \; e^{-i\vec{q}_T \cdot \vec{b}} \; S(b) \, \tilde{f}_{q/A}(x_1, C/b) \, \tilde{f}_{q/B}(x_2, C/b)$$

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- since S is of a nonperturbative origin it makes sense to include $\sqrt{S}$ into each $\tilde{f}$ PDF to get the TMD factorization formula

$$\frac{d\sigma}{dq_T^2} \sim \int dx_1 dx_2 \; d\hat{\sigma}_{qq} \int d^2 k_T \;\; \mathcal{F}_{q/A}(x_1, k_T) \, \mathcal{F}_{q/B}(x_2, |\vec{k}_T - \vec{q}_T|)$$

- the factorization is proven to leading power $\mathcal{O}(q_T/\mu)$ – no q_T flowing into the hard process (in particular, there are no off-shell hard factors)

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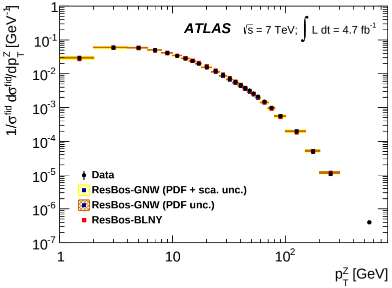
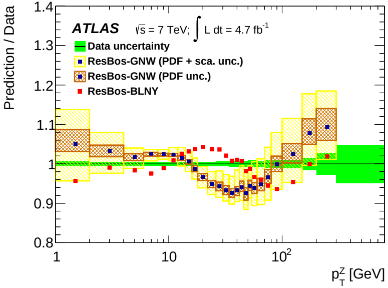




[ATLAS, JHEP09(2014)145]

Part II

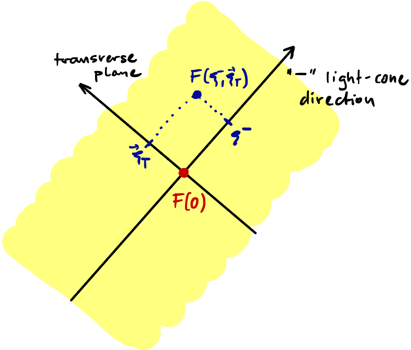
TMD gluon distributions

TMD gluon distributions

Operator definitions

Naive gluon distribution:

$$\mathcal{F}(x, k_T) \sim \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \langle P| F_a^{i+}(0) F_a^{i+}\left(\xi^+ = 0, \xi^-, \vec{\xi}_T\right)|P\rangle$$



Not gauge invariant – we need the Wilson line.

TMD gluon distributions

Operator definitions

Gauge invariant definition:

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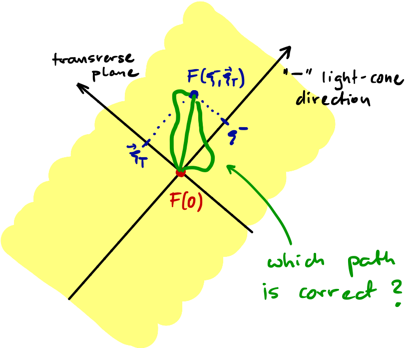

TMD gluon distributions

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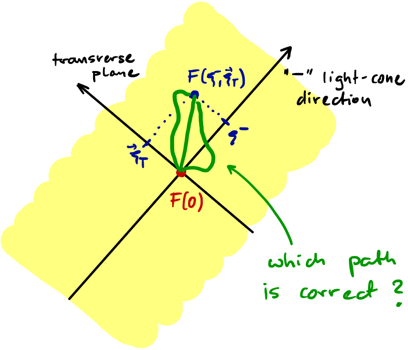
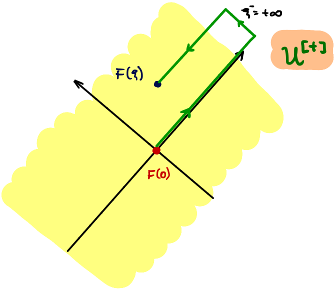


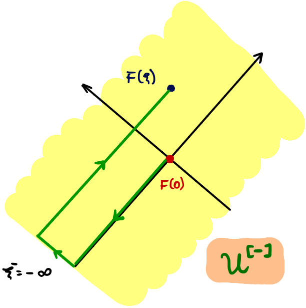
The path follows from factorization of collinear gluons.

TMD gluon distributions

Building blocks of any path (relevant to factorization)



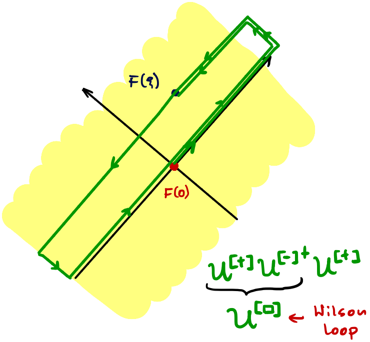
final-state interactions



initial-state interactions

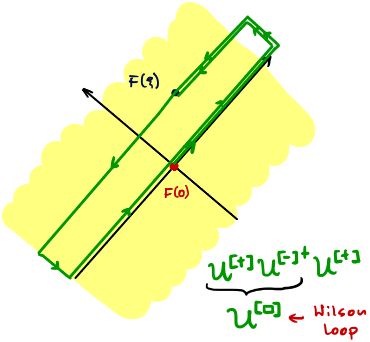
TMD gluon distributions

Example of a more complicated path



TMD gluon distributions

Example of a more complicated path



The path depends on the color structure of the hard process.

- the universality of the TMD parton distributions is lost
- for D-Y and SIDIS, for which factorization theorems are valid, the nonuniversality manifests itself as the change of sign for the polarized part

TMD gluon distributions

How 'deep' is the nonuniversality?

- The operator structures were calculated for hard processes with four [a],five [b] and six [b] colored partons.
- The gluon TMD for *any process* can be build from 10 'basis' TMD distributions [b]
- In the large N_c limit, the TMD for pure gluonic hard process with n legs was calculated for any n (in the small x limit though) [b]

[[a] C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J.C. 47, 147 (2006)]

[[b] M. Bury, PK, K. Kutak, arXiv:1809.08968]

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Although these considerations ignore the complete factorization procedure*, these operators are very useful, especially at small x.

* For more then two colored partons there will be no all-order TMD factorization theorem.

TMD gluon distributions

'Basis' TMD gluon distributions

[M. Bury, PK, K. Kutak, arXiv:1809.08968]

$$\mathcal{F}_{qg}^{(1)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(2)} \sim \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(3)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(4)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)} \sim \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(7)} \sim \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

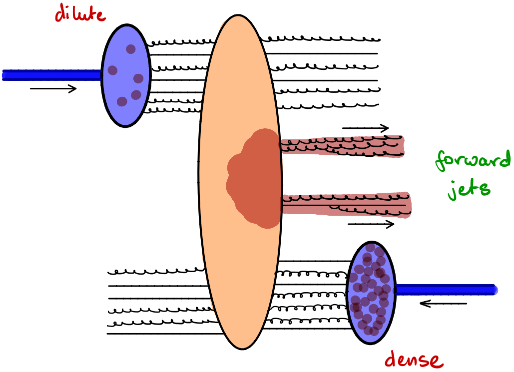
Above we use the fundamental representation for gluon field: $\hat{F}^{\mu\nu} = F_a^{\mu\nu} t^a$.

Part III

Small x TMD factorization

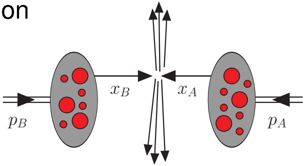
Forward jets

Basic idea



Forward jets

- central production



$x_A \simeq x_B$

Forward jets

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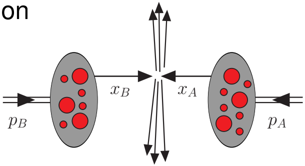

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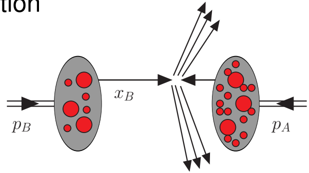

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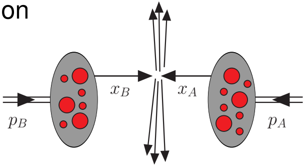

$x_A \simeq x_B$

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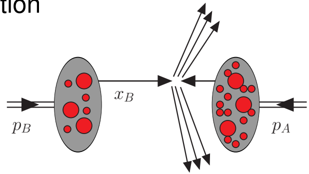
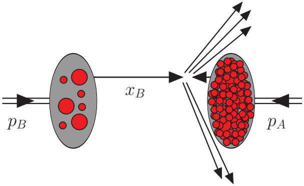


$x_A \ll x_B$

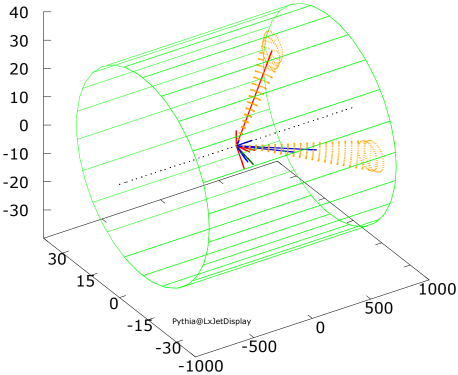
- saturation



x_A really small

Forward jets

Realistic setup at LHC



Example event (PYTHIA):

- jets with $p_{T1} \sim 27\,\mathrm{GeV}$, $p_{T2} \sim 30\,\mathrm{GeV}$
- $y_1, y_2 > 3.5$
- 9 MPI events (not all visible; each in different color)
- jet disbalance $q_T \sim 10\,\mathrm{GeV}$

Framework: Improved TMD factorization (ITMD)

Factorization formula for forward dijets in *pA*

[PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106]

$$\frac{d\sigma_{AB}}{dy_1 d^2 p_{T1} dy_2 d^2 p_{T1}} \sim \sum_{a,c,d} f_{a/B}\left(x_B, \mu^2\right) \sum_{i=1,2} \Phi^{(i)}_{ag\to cd}\left(x_A, k_T^2, \mu^2\right) K^{(i)}_{ag\to cd}(k_T, \mu^2)$$

$f_{a/B}$ – collinear PDFs in proton

$\Phi^{(i)}_{ag\to cd}$ – TMD gluon distributions in nucleus for $ag \to cd$

$K^{(i)}_{ag\to cd}$ – off-shell gauge invariant hard factors

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$K^{(i)}_{ag\to cd}$ – off-shell gauge invariant hard factors

$$\Phi^{(1)}_{qg\to gq} = \mathcal{F}^{(1)}_{qg}, \qquad \Phi^{(2)}_{qg\to gq} = \frac{1}{N_c^2-1}\left(N_c^2 \mathcal{F}^{(2)}_{qg} - \mathcal{F}^{(1)}_{qg}\right),$$

$$\Phi^{(1)}_{gg\to q\bar{q}} = \frac{1}{N_c^2-1}\left(N_c^2 \mathcal{F}^{(1)}_{gg} - \mathcal{F}^{(3)}_{gg}\right), \qquad \Phi^{(2)}_{gg\to q\bar{q}} = \mathcal{F}^{(3)}_{gg} - N_c^2 \mathcal{F}^{(2)}_{gg},$$

$$\Phi^{(1)}_{gg\to gg} = \frac{1}{2N_c^2}\left(N_c^2 \mathcal{F}^{(1)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2 \mathcal{F}^{(6)}_{gg}\right),$$

$$\Phi^{(2)}_{gg\to gg} = \frac{1}{N_c^2}\left(N_c^2 \mathcal{F}^{(2)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2 \mathcal{F}^{(6)}_{gg}\right)$$

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- formula resumes power corrections $\mathcal{O}(k_T/\mu)$, where the hard scale $\mu \sim$ average transverse momentum of jets
- when $Q_s \sim k_T \ll \mu$ it coincides with Color Glass Condensate (CGC) theory
- when $Q_s \ll k_T \sim \mu$ it coincides with the ordinary k_T factorization
- power-by-power comparison with CGC is under study...
- implemented in the MC code

TMD gluon distributions

How to get the TMD gluon distributions?

At small x the TMD gluon distributions can be identified with the quantities that appear in the Color Glass Condensate (CGC) effective theory.

This allows for at least three options:

1. use a rough model of a nucleus – the McLarren-Venugopalan model
2. use the B-JIMWLK evolution equation known in CGC

   [C. Marquet, E. Petreska, C. Roiesnel, JHEP 1610 (2016) 065]

   [collaboration with P. Korcyl and K. Cichy on use of the lattice methods]

3. in the large N_c limit, there are 6 contributing TMD distributions; in addition, in certain mean field approximation all of them can be calculated from just one independent distribution, so called dipole gluon distribution $\mathcal{F}_{qg}^{(1)}$
   - it can be taken from the Golec-Biernat-Wusthoff model

     [E. Petreska, Nucl.Phys. A956 (2016) 894-897]

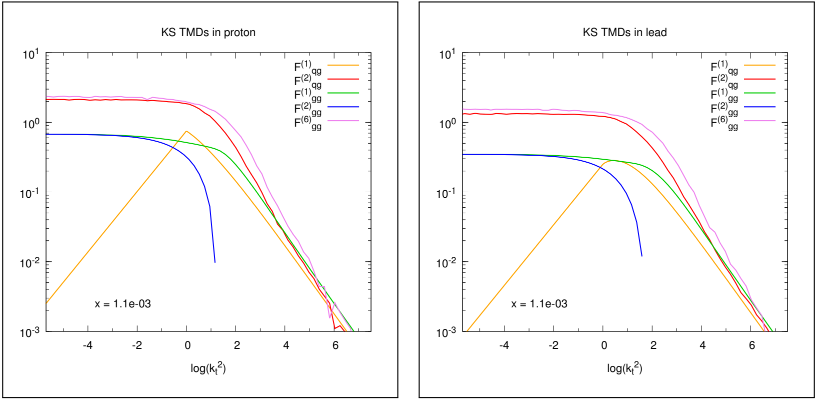
   - it can be fitted to data assuming certain nonlinear evolution equation

     [K. Kutak, S. Sapeta, Phys. Rev. D 86 (2012) 094043]

     [A. van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta]

TMD gluon distributions

The dipole gluon was fitted to HERA data by Kutak and Sapeta[1] (KS) using rather involved nonlinear evolution equation proposed by Kutak-Kwieciński[2]. [A. van Hameren, P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, JHEP 1612 (2016) 034]



All gluons merge for large k_T (except $\mathcal{F}_{gg}^{(2)}$ which vanishes) $\Rightarrow$ correct HEF limit.

[1] K. Kutak, S. Sapeta, Phys. Rev. D 86 (2012) 094043 [2] K. Kutak, K. Kwiecinski, Eur. Phys. J. C 29 (2003) 521

Part IV

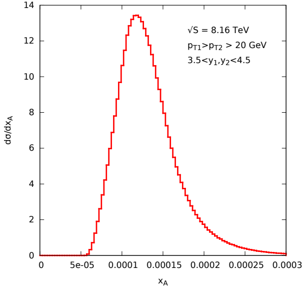
Applications to LHC physics

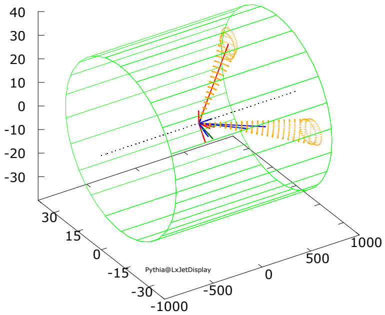
Results for dijet production in *pPb* at LHC

Kinematic cuts

- CM energy: $\sqrt{S} = 8.16\,\mathrm{TeV}$
- require two jets with $(\Delta\phi)^2 + (\Delta\eta)^2 > R^2, R = 0.5$
- transverse momenta cuts: $p_{T1} > p_{T2} > 20\,\mathrm{GeV}$
- rapidity cuts: $3.5 < y_1, y_2 < 4.5$

x fractions probed





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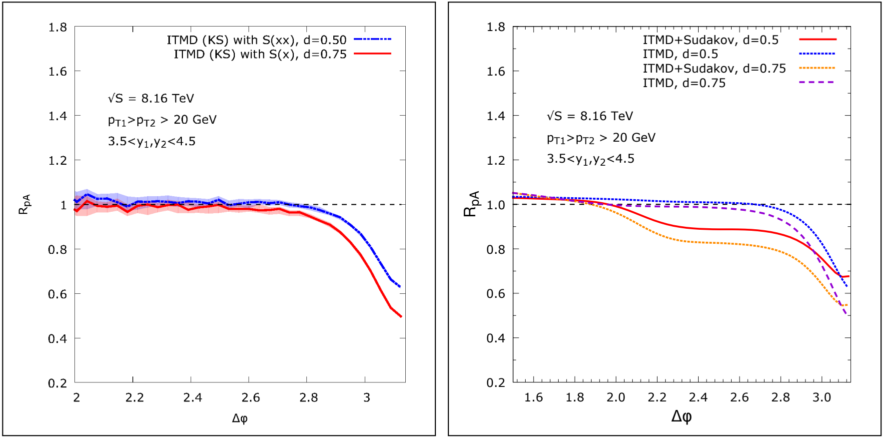
Azimuthal decorrelations

[A. van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, JHEP 1612 (2016) 034]

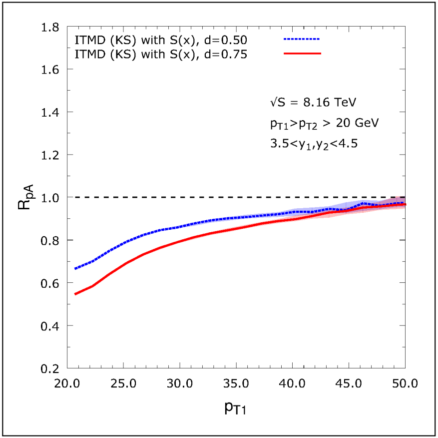


Results for dijet production in *pPb* at LHC

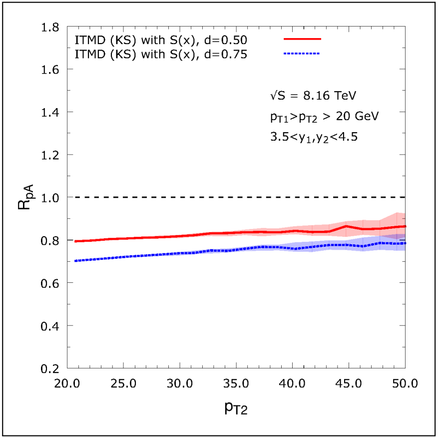
Jet p_T spectra

[A. van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, JHEP 1612 (2016) 034]

leading jet spectrum



subleading jet spectrum:



Digression: two basic TMD gluon distributions

In general, there are two most basic TMD gluon distributions, which seem to be fundamental:

[F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan Phys.Rev. D 83 (2011) 105005]

1. dipole gluon distribution

$$\mathcal{F}_{qg}^{(1)} \sim \langle P|\, \mathrm{Tr}\{F(\xi)\, \mathcal{U}^{[-]\dagger} F(0)\, \mathcal{U}^{[+]}\}\, |P\rangle$$

appears directly in: inclusive DIS, inclusive jet in *pA*

2. Weizsacker-Williams (WW) gluon distribution

$$\mathcal{F}_{gg}^{(3)} \sim \langle P|\, \mathrm{Tr}\left\{F(\xi)\, \mathcal{U}^{[+]\dagger} F(0)\, \mathcal{U}^{[+]}\right\} |P\rangle$$

appears directly in dijets in γA $\Rightarrow$ can be studied in UPC

Dijet production in UPC at LHC

Improved TMD factorization for $\gamma A \to 2j + X$

$$\frac{d\sigma_{\gamma A\to 2j}}{dy_1 d^2p_{T1} dy_2 d^2p_{T1}} \sim \mathcal{F}_{gg}^{(3)}\left(x_A, k_T^2, \mu^2\right) \otimes K_{\gamma g^*\to q\bar{q}}\left(k_T, \mu^2\right)$$

$\mathcal{F}_{gg}^{(3)}$ the WW gluon distribution

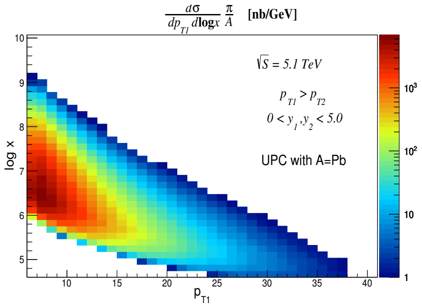
$K_{\gamma g^*\to q\bar{q}}$ off-shell hard factor for the $\gamma g^* \to q\bar{q}$ process

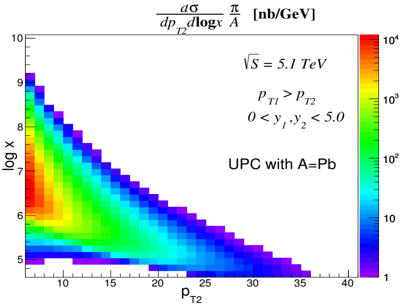
- Formula is as simple as for e.g. inclusive DIS, but probes different gluon distribution
- For UPC one needs to convolute this with the photon flux from nucleus (equivalent photon approximation)
- In UPC, the problem is that the photon flux dies out very fast above $x_\gamma \sim 0.03$ for Pb, so there is not much 'space' for the asymmetric kinematics $x_A \ll x_\gamma$ at current LHC energies with reasonable p_T cuts.

Results for dijets in UPC at LHC

Kinematic cuts

CM energy: 5.1 TeV	rapidity: $0 < y_1, y_2 < 5$
transverse momenta: $p_{T1} > p_{T2} > p_{T0}$, $p_{T0} = 6 \div 25\,\mathrm{GeV}$	jet algorithm: $R = 0.5$





Results for dijets in UPC at LHC

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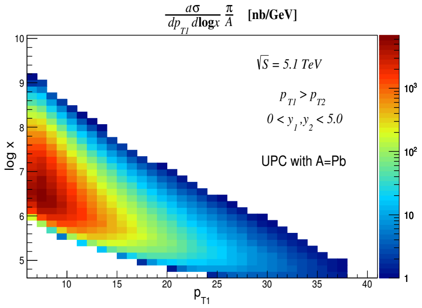

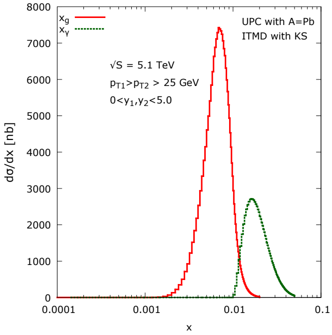

Results for dijets in UPC at LHC

Kinematic cuts

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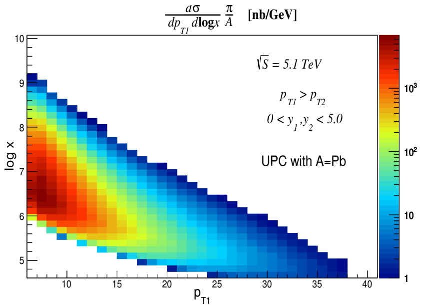

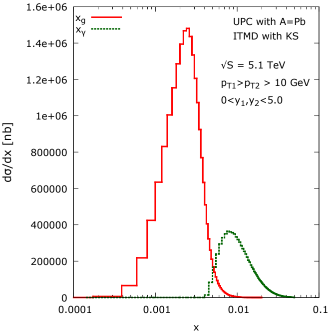

Results for dijets in UPC at LHC

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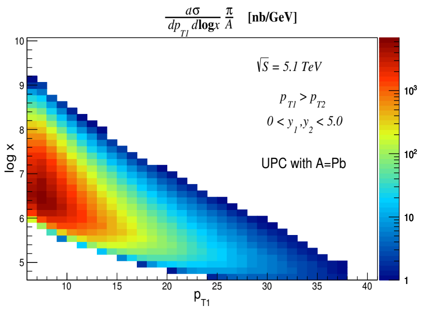
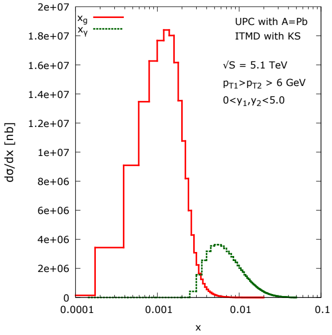




Results for dijets in UPC at LHC

Nuclear modification factor $R_{\gamma A}$

For UPC collisions we define the nuclear modification ratio as

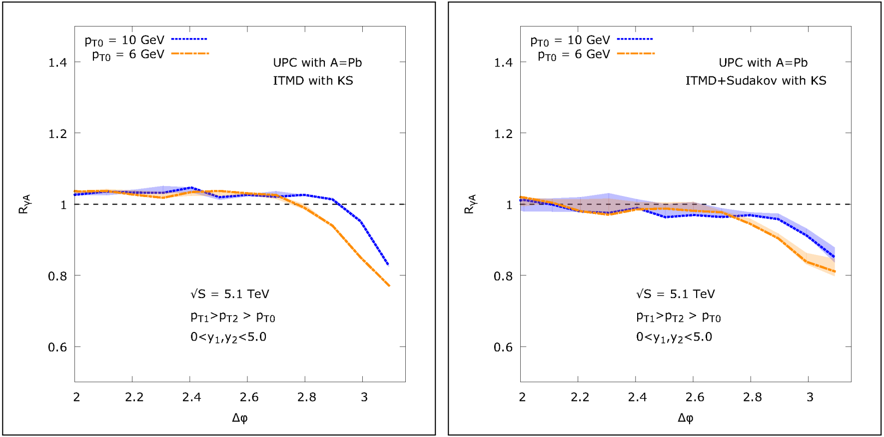
$$R_{\gamma A} = \frac{d\sigma_{AA}^{\mathrm{UPC}}}{A d\sigma_{Ap}^{\mathrm{UPC}}}$$

where $A = \mathrm{Pb}$ and the $d\sigma_{Ap}^{\mathrm{UPC}}$ is with jets going in the nucleus direction.

Results for dijets in UPC at LHC

Azimuthal decorrelations

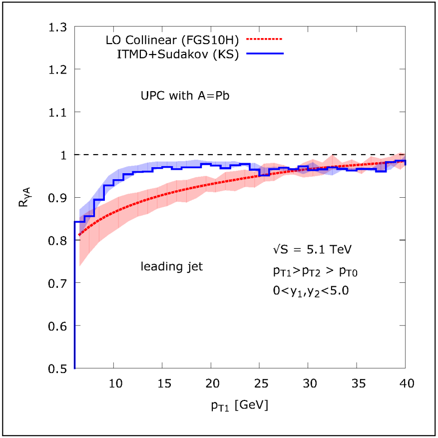
[PK, K. Kutak, S. Sapeta, A. Stasto, M. Strikman, Eur.Phys.J. C77 (2017) no.5, 353]

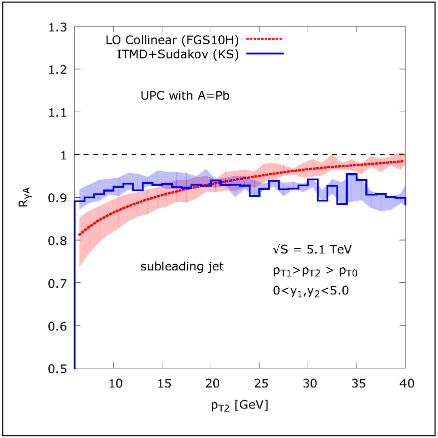


Results for dijets in UPC at LHC

Jet p_T spectra

[PK, K. Kutak, S. Sapeta, A. Stasto, M. Strikman, Eur.Phys.J. C77 (2017) no.5, 353]





Summary & Outlook

Summary

- Forward jet production, taking into account gluon saturation, can be formulated using the TMD factorization-like framework
- Main complication: several process dependent gluon distributions
- Two basic gluon distributions: dipole and WW
- Dijets UPC is a clean probe for WW gluon distribution
- Pheno predicts large suppression of nuclear modification ratio for pA, around 35-40%
- For UPC the suppression is rather small, but visible, up to 20%

We obviously need:

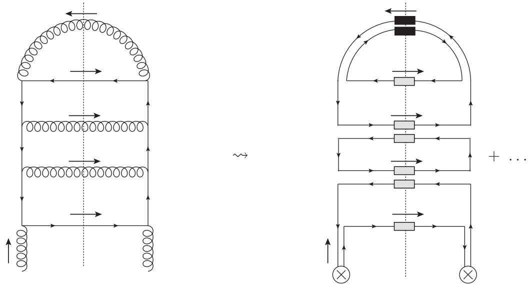
- Better TMD gluon distributions, ideally from B-JIMWLK with kinematic constraint
- NLO computations...

BACKUP

TMD gluon distributions

Example: TMD for a particular diagram

[M. Bury, PK, K. Kutak, arXiv:1809.08968]



$$N_c \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}\,\mathrm{Tr}\mathcal{U}^{[\square]}\,\mathrm{Tr}\mathcal{U}^{[\square]\dagger}$$

where Wilson lines and loops are defined as

$$\mathcal{U}^{[\pm]} = U(0,\pm\infty;0_T)U(\pm\infty,\xi^+;\xi_T) \quad \mathcal{U}^{[\square]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

Off-shell gauge invariant amplitudes

Color decomposition of gluon amplitudes:

$$\mathcal{M}^{a_1 \ldots a_N}\left(\varepsilon_1^{\lambda_1}, \ldots, \varepsilon_N^{\lambda_N}\right) = \sum_{\sigma \in S_N} \mathrm{Tr}\left(t^{a_{\sigma_1}} t^{a_{\sigma_2}} \ldots t^{a_{\sigma_N}}\right) \mathcal{M}\left(\sigma_1^{\lambda_{\sigma_1}}, \sigma_2^{\lambda_{\sigma_2}} \ldots, \sigma_N^{\lambda_{\sigma_N}}\right)$$

a_i- color indices, $\varepsilon_i^{\lambda_i}$ - polarization vectors with helicity λ_i, S_N - set of noncyclic permutations.

In spinor formalism, the non-zero off-shell helicity amplitudes have the form of the MHV amplitudes with certain modification of spinor products:

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029]

$$\mathcal{M}_{g^* g \to gg}\left(1^*, 2^-, 3^+, 4^+\right) = 2g^2 \rho_1 \frac{\langle 1^* 2\rangle^4}{\langle 1^* 2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^* g \to gg}\left(1^*, 2^+, 3^-, 4^+\right) = 2g^2 \rho_1 \frac{\langle 1^* 3\rangle^4}{\langle 1^* 2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^* g \to gg}\left(1^*, 2^+, 3^+, 4^-\right) = 2g^2 \rho_1 \frac{\langle 1^* 4\rangle^4}{\langle 1^* 2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

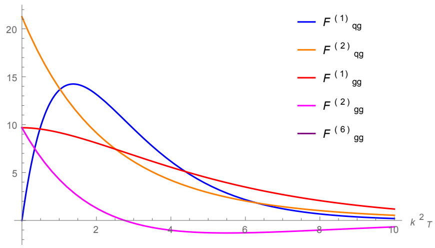
where $\langle ij\rangle = \langle k_i - | k_j + \rangle$ with spinors defined as $|k_i \pm\rangle = \frac{1}{2}\left(1 \pm \gamma_5\right) u\left(k_i\right)$. Spinor products for off-shell states involve only longitudinal component of the off-shell momentum $\langle 1^* i\rangle = \langle p_1 i\rangle$, where $k_1 = p_1 + k_{T1}$, $k_1^2 \neq 0$, $p_1^2 = 0$.

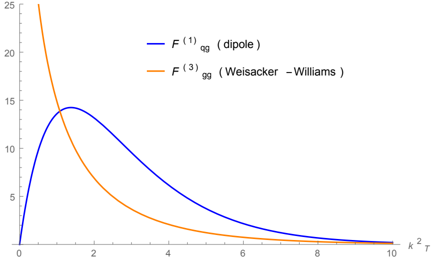
TMD gluon distributions: GBW model

The Golec-Biernat-Wusthoff (GBW) model:

$$\mathcal{F}_{qg}^{(1)}\left(x, k_T^2\right) = \frac{N_c S_\perp}{2\pi^3 \alpha_s} \frac{k_T^2}{Q_s^2(x)} \exp\left(-\frac{k_T^2}{Q_s^2(x)}\right), \quad Q_s(x) = Q_{s0}^2 \left(\frac{x}{x_0}\right)^\lambda$$

Assuming gaussian distribution of colour sources all five gluons can be calculated analytically[1]





[1] E. Petreska, Proceedings, 7th International Workshop MPI@LHC 2015