Quantum Kibble-Zurek mechanism: scaling hypothesis in the Ising and Bose-Hubbard models

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Phys. Rev. B 2016

Phys. Rev. B (2017)

review on quantum KZM:
JD, Adv. in Phys. 59, 1063 (2010)

Quantum Ising Chain

$$H = -\sum_{n=1}^{N} \left(g\sigma_n^z + \sigma_n^x \sigma_{n+1}^x \right)$$

Strong transverse field g>>1

$$\left| \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \ldots \right\rangle$$

Quantum phase transition at g=1

Energy gap $\Delta \rightarrow 0$

Correlation length $\xi \rightarrow \infty$

Ferromagnetic ground states at g=0

$$\left| \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \ldots \right\rangle \text{ or } \left| \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \ldots \right\rangle$$

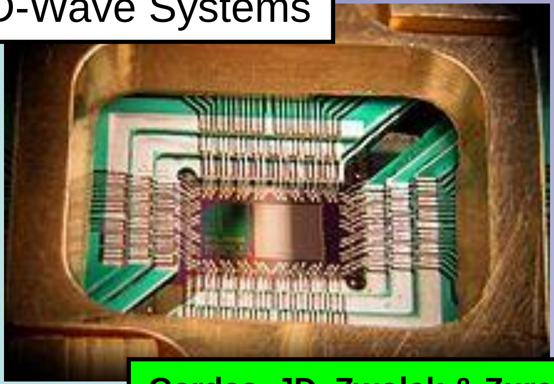
Ideal Adiabatic Quantum State Preparation (or Adiabatic Quantum Computation)

Simple H_i

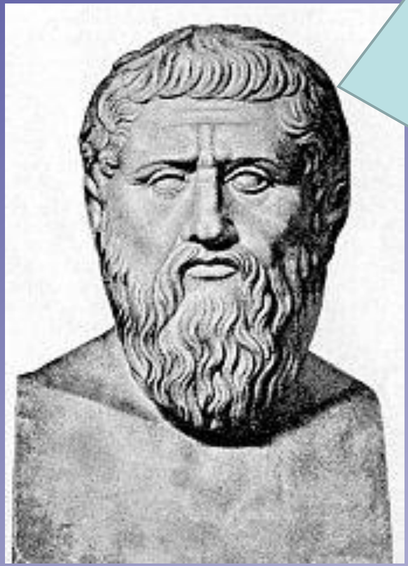
Adiabatic

H_f Interesting

D-Wave Systems



Gardas, JD, Zwolak & Zurek in preparation



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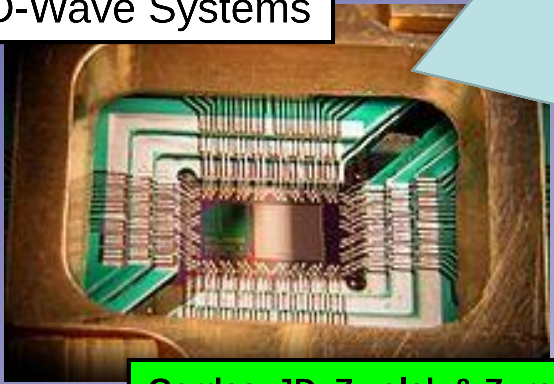
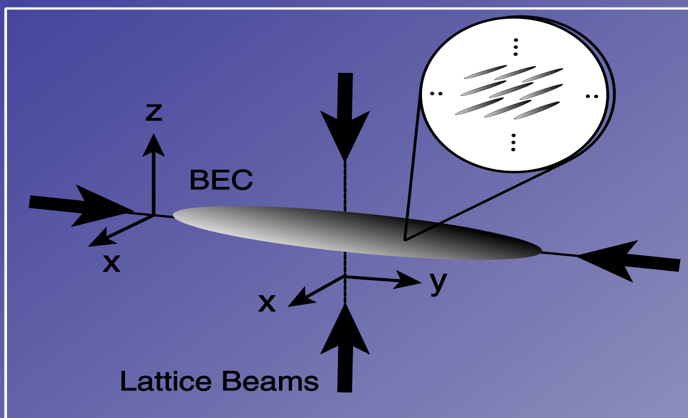
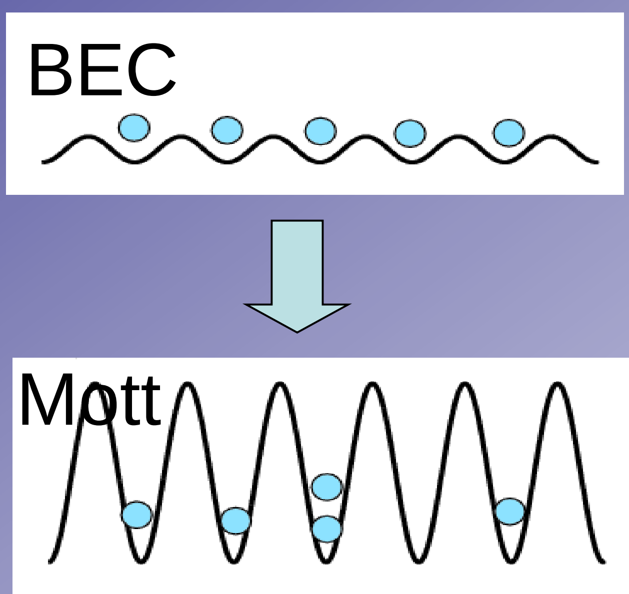


Gardas, JD, Zwolak & Zurek in preparation

Real Adiabatic Quantum State Preparation





Simple H_i

$$\text{Simple} \neq \text{Interesting}$$
$$\Downarrow$$
$$\text{Quantum Phase Transition}$$

Non-adiabatic

H_f Interesting

Quantum Ising Chain

$$H = -\sum_{n=1}^{N} \left(g\sigma_n^z + \sigma_n^x \sigma_{n+1}^x \right)$$

``Simple''

$$\left| \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \ldots \right\rangle$$

Adiabatic

Non-adiabatic

$$\left| \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \ldots \right\rangle$$

or/and/ +

$$\left| \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \ldots \right\rangle$$

``Interesting''

$$\left| \uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow \ldots \right\rangle$$

$$\hat{\xi} = ?$$

Excited

Quantum Kibble-Zurek mechanism (KZM)

$$\varepsilon = \frac{g_c - g}{g_c}$$

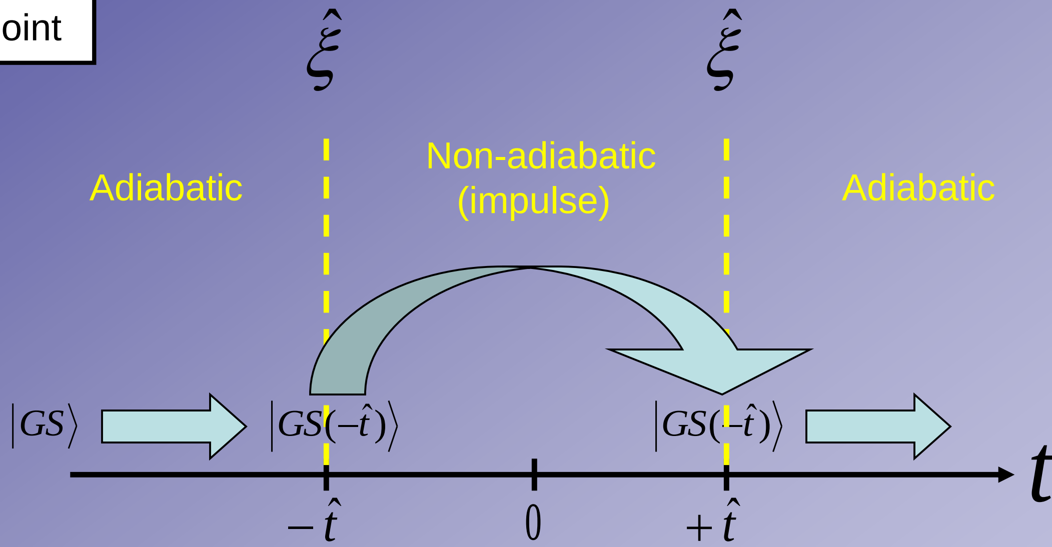
distance from the critical point

$$\Delta \propto |\varepsilon|^{z\nu}$$

energy gap

$$\xi \propto |\varepsilon|^{-\nu}$$

correlation length

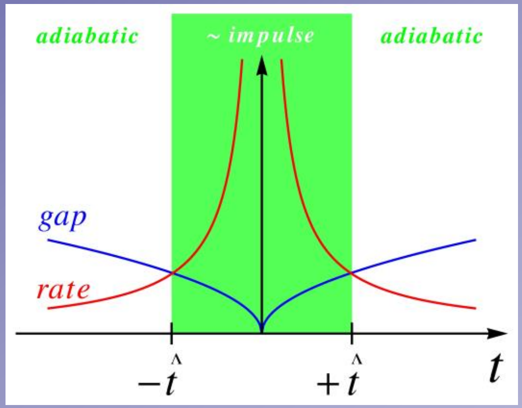


linear(ized) quench

$$\varepsilon = \frac{t}{\tau_Q}$$

$$\left|\frac{d\varepsilon/dt}{\varepsilon}\right| = \frac{1}{|t|}$$

transition rate



$$\text{RATE} = \text{GAP} \quad \text{at} \quad t = -\hat{t}$$

$$\text{where } \hat{t} \propto \tau_Q^{\frac{z\nu}{1+z\nu}} \text{ and } \hat{\xi} \propto \tau_Q^{\frac{\nu}{1+z\nu}}$$

$$|\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\ldots\rangle$$

Final excited state

$$\hat{\xi} \propto \sqrt{\tau_Q}$$

Quantum Ising chain

$$\hat{\xi} \propto \sqrt{\tau_Q} \quad \text{and} \quad \hat{t} \propto \sqrt{\tau_Q}$$

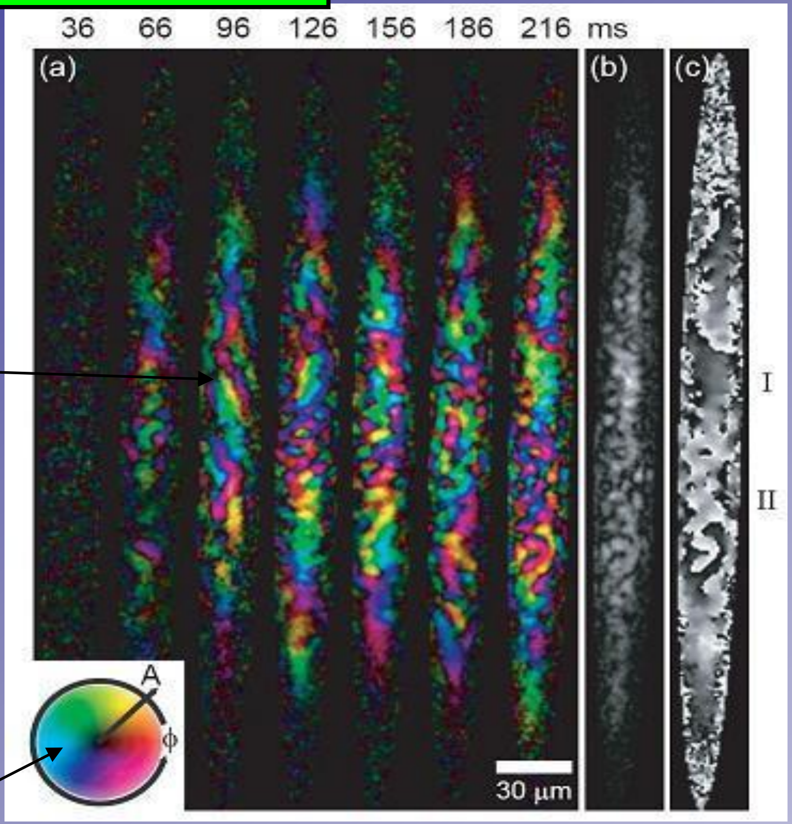
Experiment: S=1 condensate

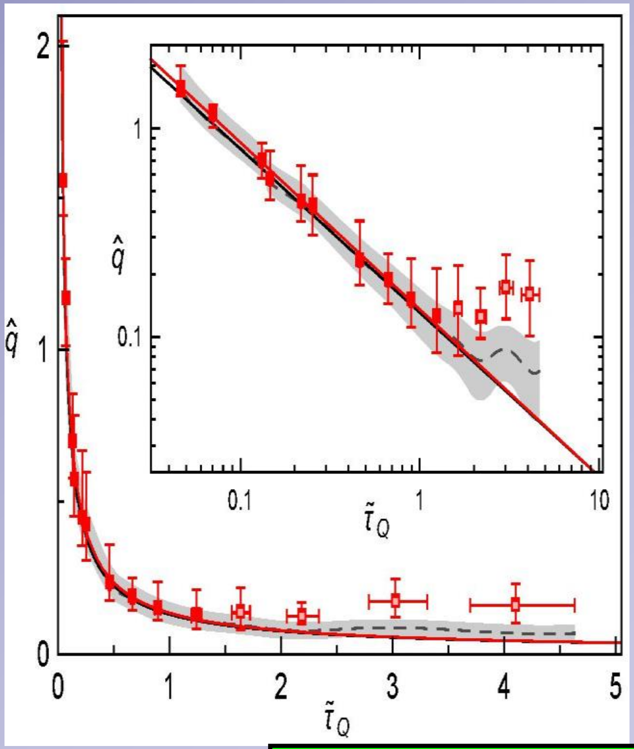
PARA → FERRO

Sadler et al., Nature 2006

Finite ferromagnetic domains

Planar spin orientation



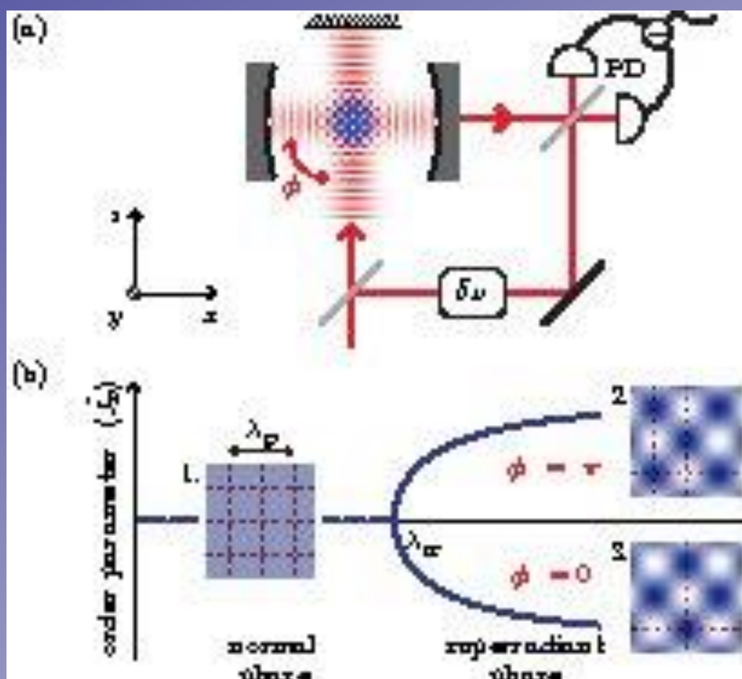


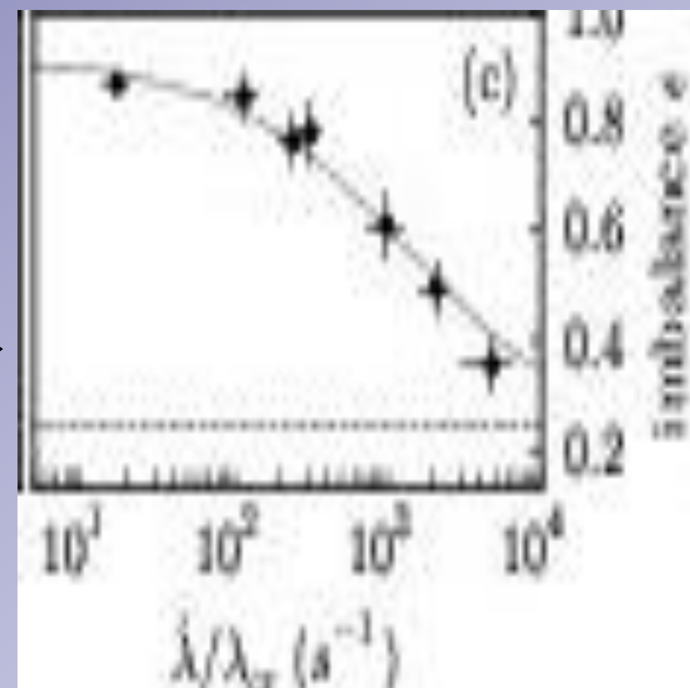
Anquez et al., PRL 2016

Symmetry breaking in the Dicke model

$$H = \hbar\omega_0 J_z + \hbar\omega a^+ a + g(a^+ + a) J_x + bias\, J_x$$

$$g > g_c : \quad \langle J_x \rangle > 0 \text{ and } \langle \text{a} \rangle < 0 \quad \text{OR} \quad \langle J_x \rangle < 0 \text{ and } \langle \text{a} \rangle > 0$$



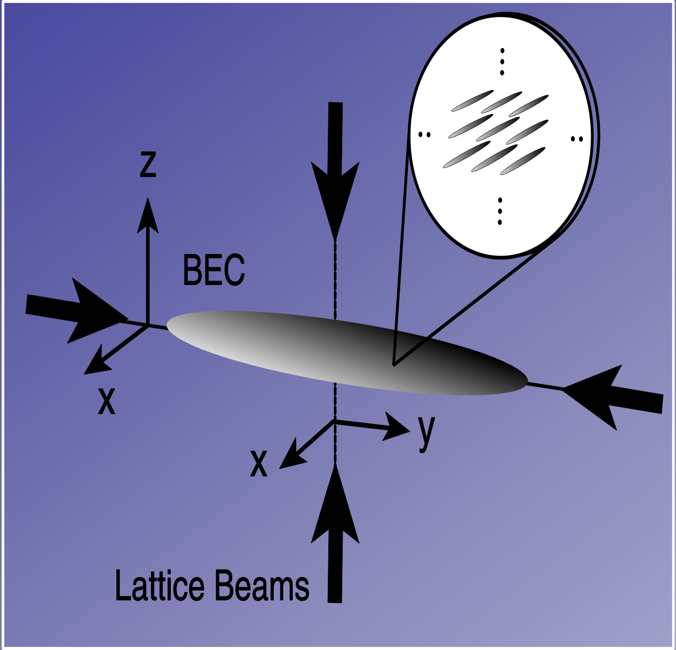


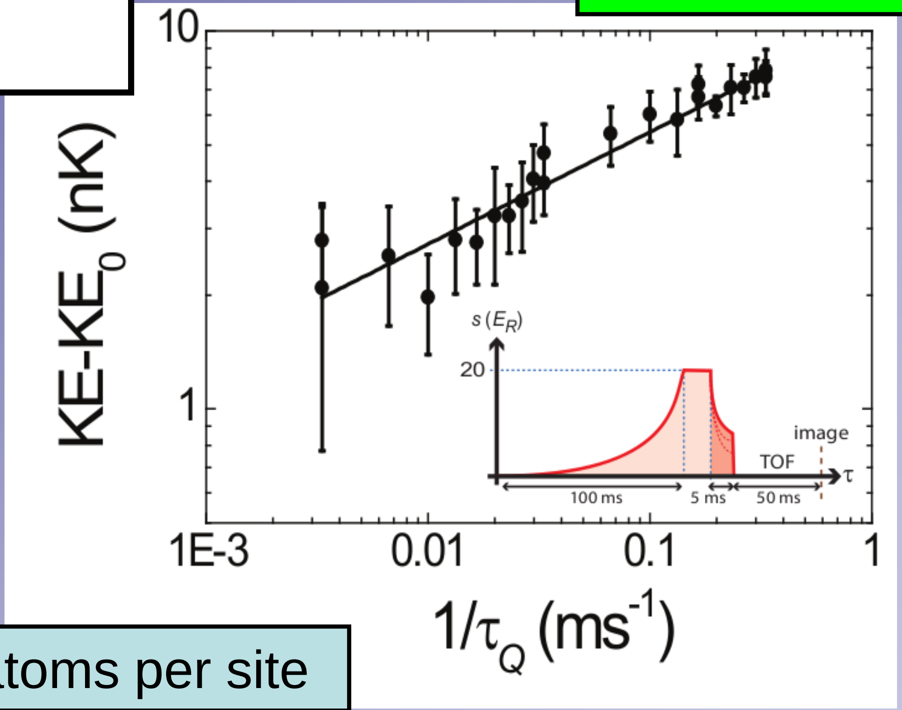
Esslinger's group, PRL 2011

Mott -> superfluid transition in 3D (Bose-Hubbard Model)

$$E_{exc} \approx \tau_Q^{-0.31...-0.32}$$

Experiment: DeMarco et al. PRL 2011, Nature Ph. 2015





n=3,4 atoms per site

n=1 atom per site

Experiment Schneider et al. PNAS 2014

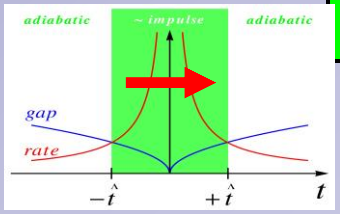
???

Theory

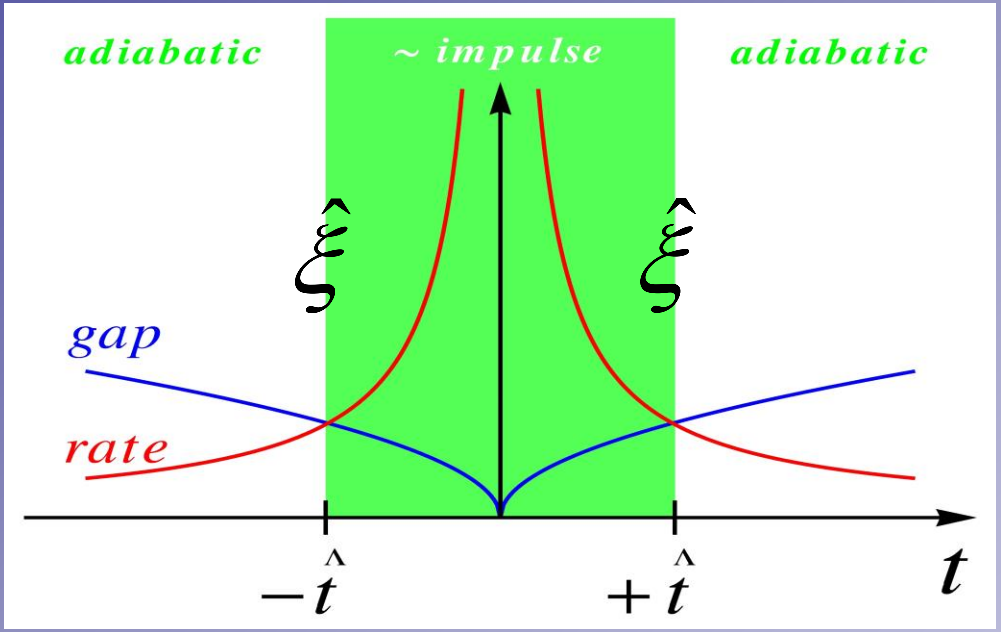
$$E_{exc} \approx \tau_Q^{-1/3}$$

n infinite

Meisner, JD, Zurek PRL 2008



K-Z scaling hypothesis



scaled time

$$t/\hat{t}$$

scaled distance

$$x/\hat{\xi}$$

scales diverge in the adiabatic limit

unique scales in
long wavelength & low frequency regime

they are not independent

$$\hat{t} \propto \hat{\xi}^{z}$$

scaling dimension

$$\langle \psi(t) | \hat{O}(x) | \psi(t) \rangle = \hat{\xi}^{-\Delta_O} F_O(x/\hat{\xi}, t/\hat{\xi}^{z})$$

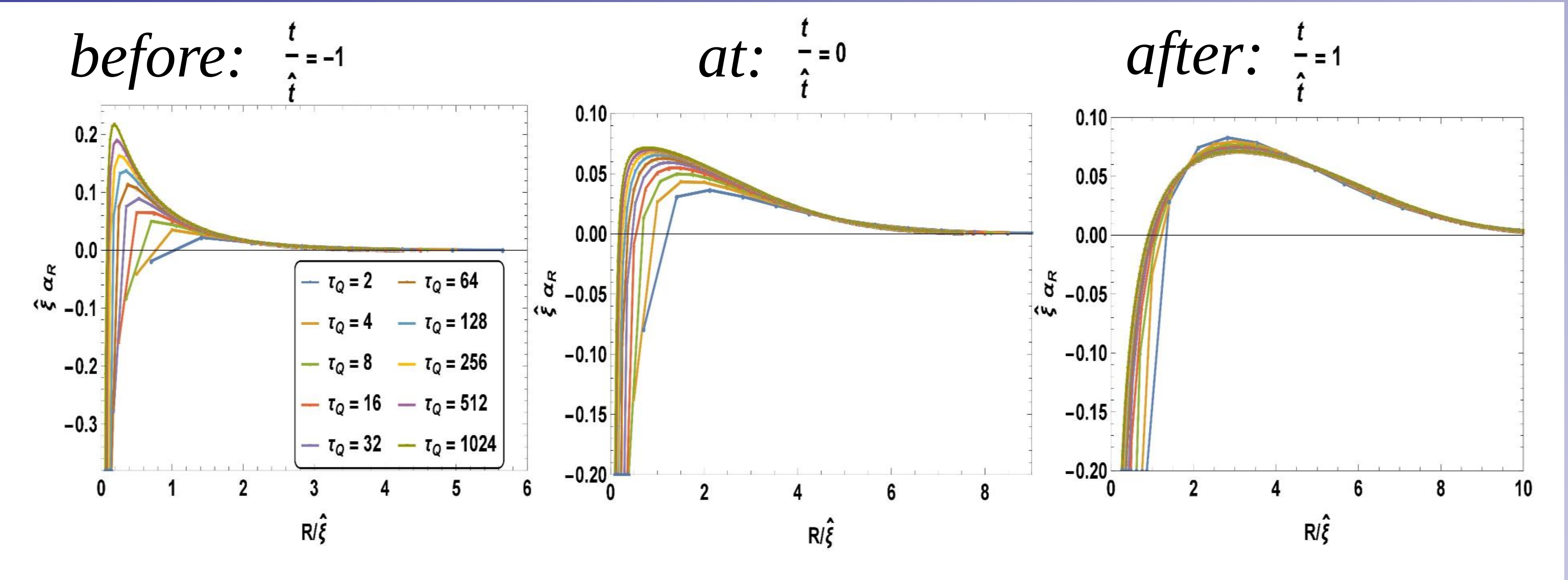
scaling function

Jordan-Wigner transformation

$$\sigma_n^x, \sigma_n^z \to c_n$$

quadratic correlator

$$\alpha_R = \langle c_{n+R} c_n^+ \rangle$$



$$\alpha_R = \hat{\xi}^{-1} F_\alpha (R / \hat{\xi}, t / \hat{\xi})$$

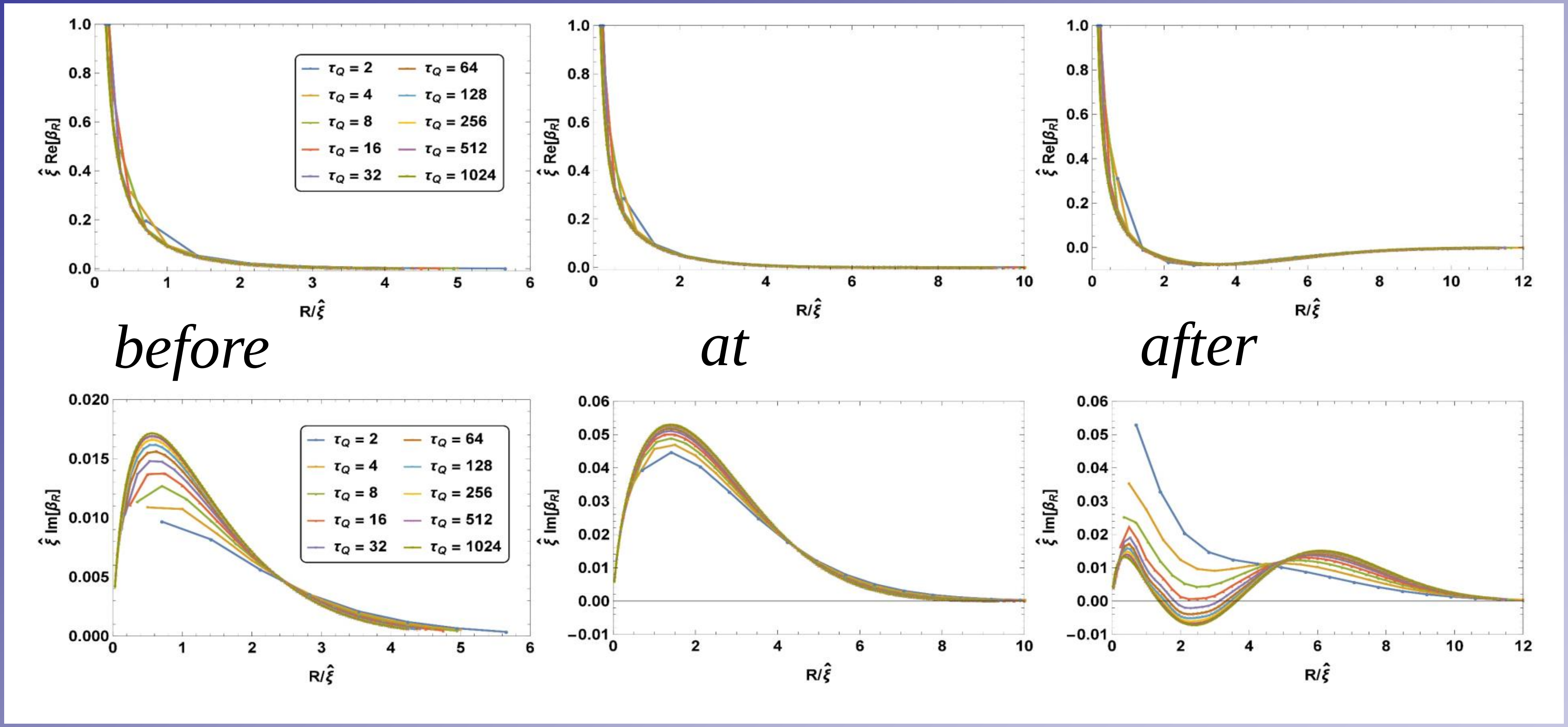
$$\hat{\xi} = \sqrt{\tau_Q}$$

Jordan-Wigner transformation

$$\sigma_n^x, \sigma_n^z \rightarrow c_n$$

anomalous correlator

$$\beta_R = \langle c_{n+R} c_n \rangle$$

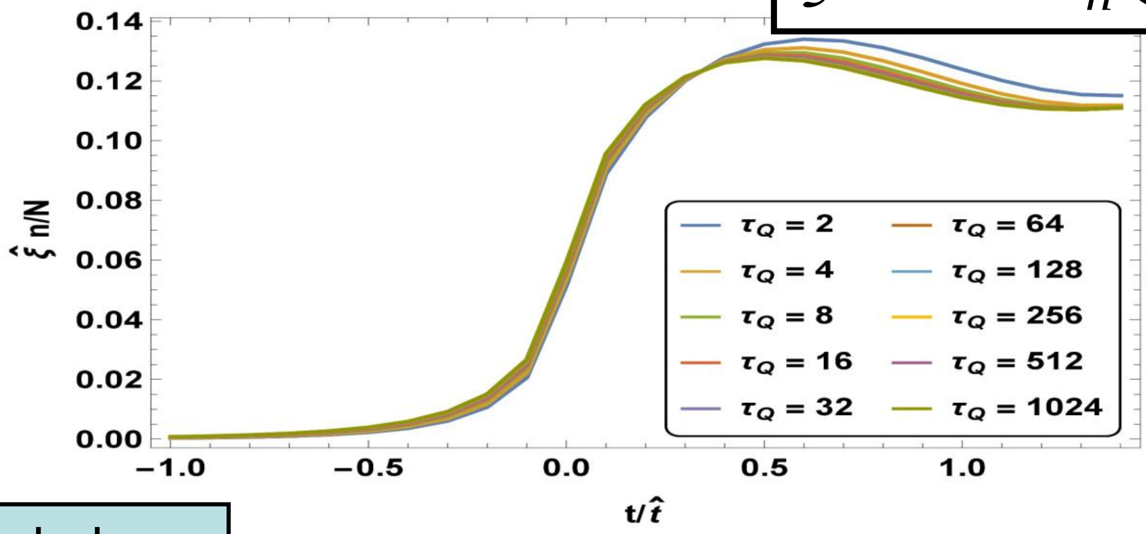


$$\hat{\xi}^1 \beta_R = F_\beta(R/\hat{\xi}, t/\hat{\xi})$$

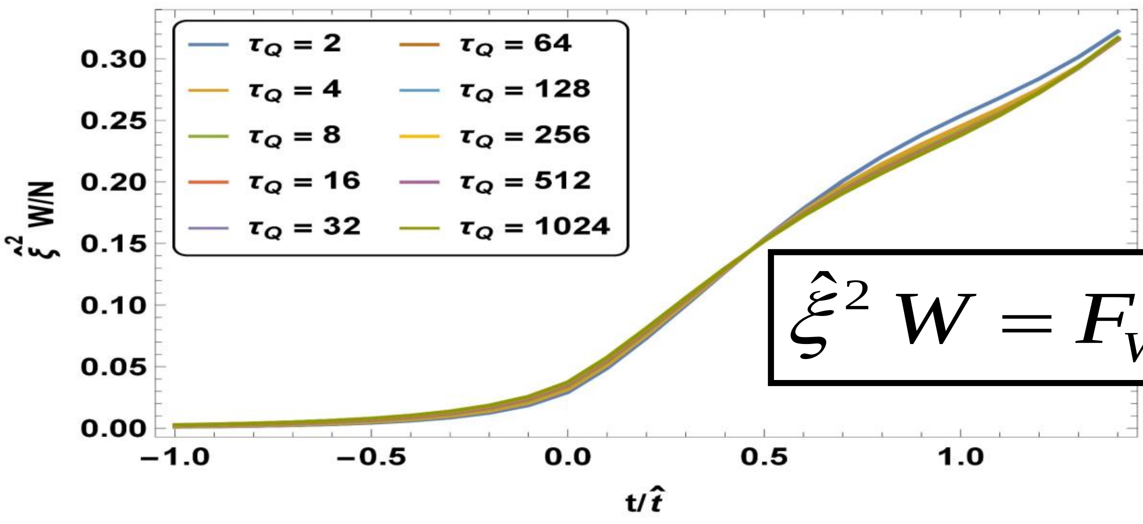
$$\hat{\xi} = \sqrt{\tau_Q}$$

density of quasiparticles

$$\hat{\xi}^{1}\, n = F_{n}(t\,/\,\hat{\xi})$$



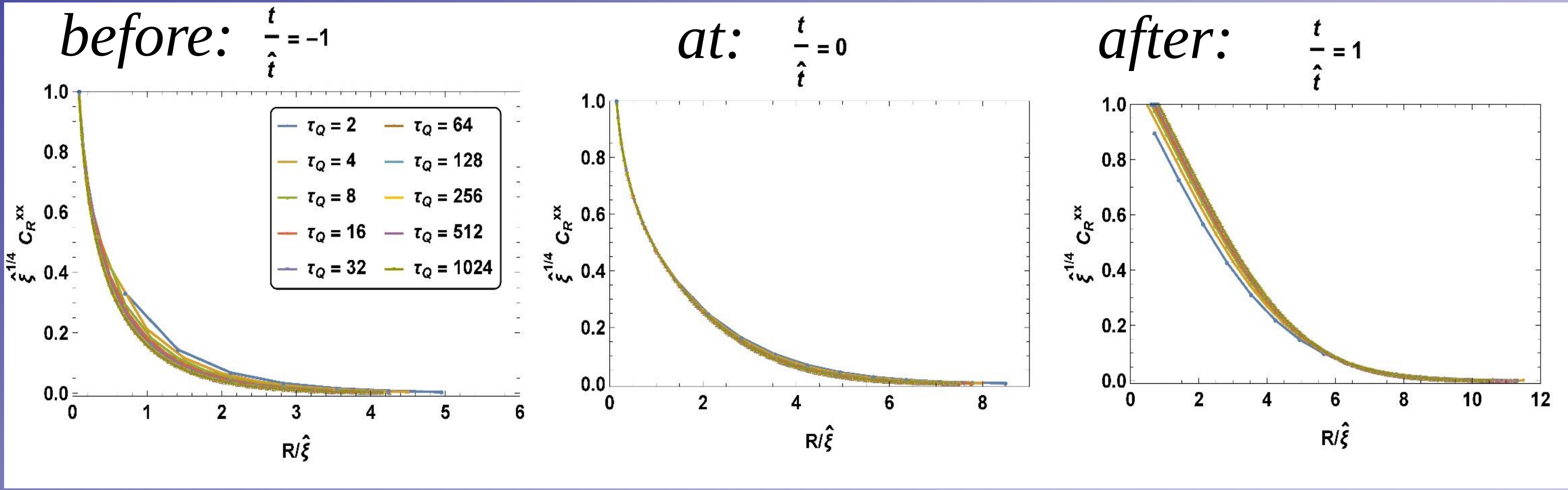
density of work done



$$\hat{\xi}^{2}\, W = F_{W}(t\,/\,\hat{\xi})$$

ferromagnetic correlator

$$C_R^{xx} = \langle \sigma_{n+R}^x \sigma_n^x \rangle - \langle \sigma_{n+R}^x \rangle \langle \sigma_n^x \rangle$$



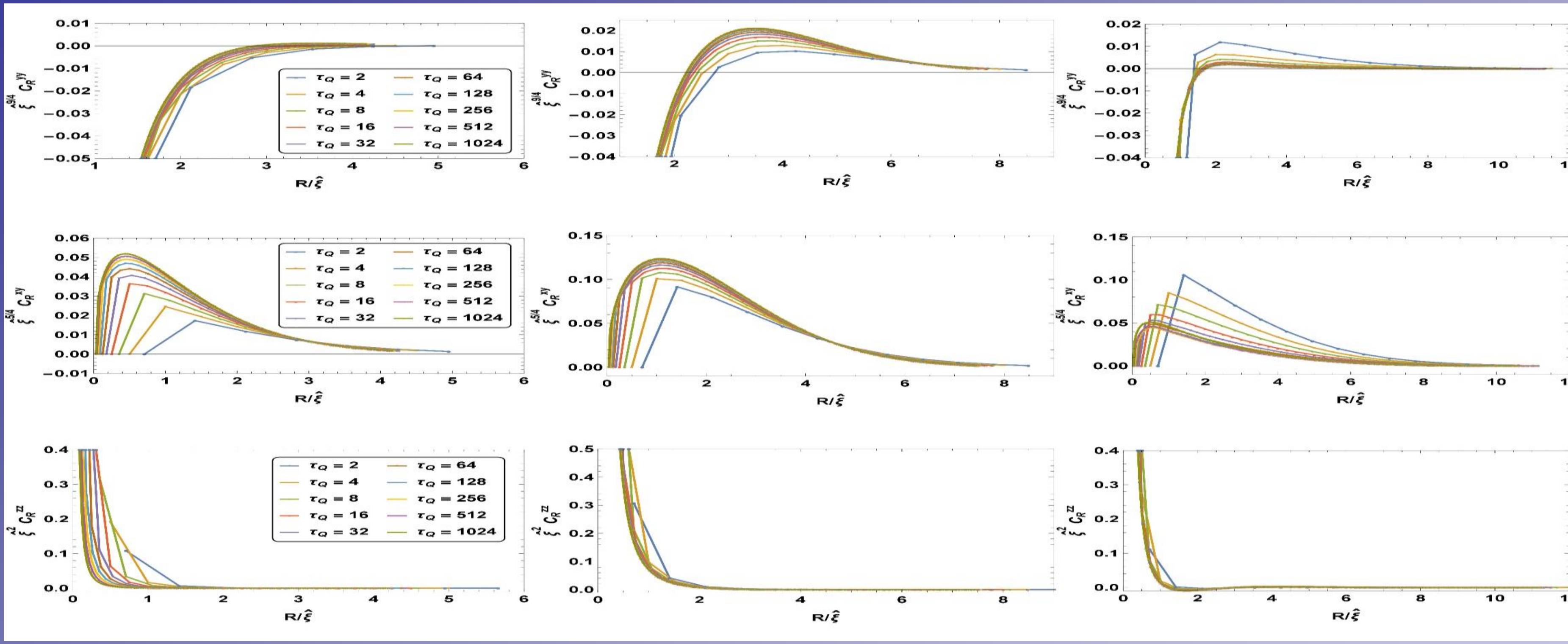
$$\hat{\xi}^{1/4}\, C_{xx}(R) = F_{xx}(R/\hat{\xi}, t/\hat{\xi})$$

$$\hat{\xi} = \sqrt{\tau_Q}$$

see also M. Kolodrubetz

more correlation functions

$$C_R^{ab} = \left\langle \sigma_{n+R}^{a} \sigma_{n}^{b} \right\rangle - \left\langle \sigma_{n+R}^{a} \right\rangle \left\langle \sigma_{n}^{b} \right\rangle$$



$$\hat{\xi}^{9/4}\, C_R^{yy} = F_{yy}(R/\hat{\xi}, t/\hat{\xi})$$

$$\hat{\xi} = \sqrt{\tau_Q}$$

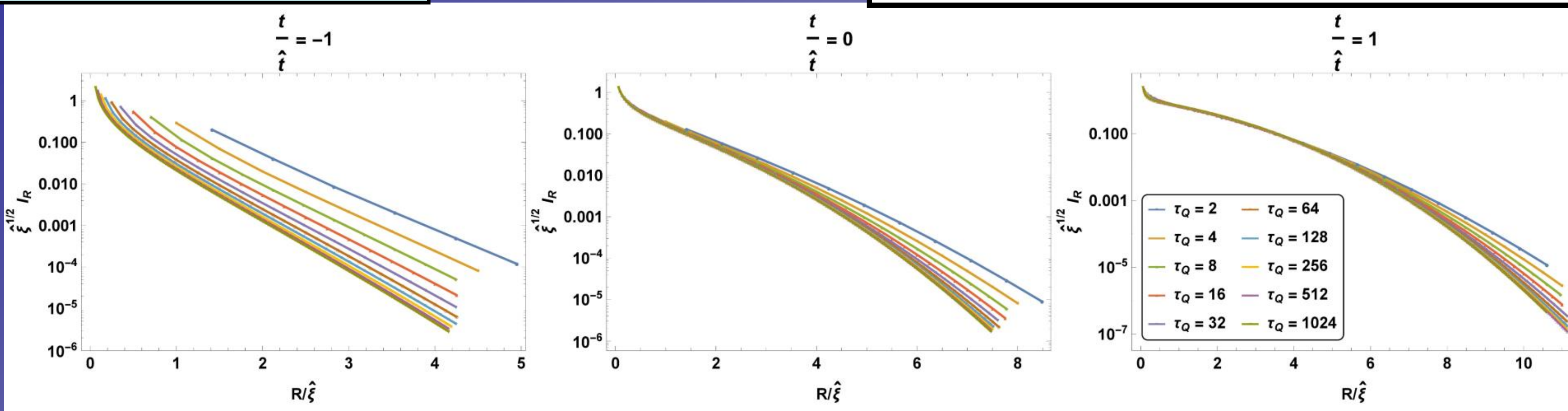
$$\hat{\xi}^{5/4}\, C_R^{xy} = F_{xy}(R/\hat{\xi}, t/\hat{\xi})$$

$$\hat{\xi}^{1}\, C_R^{zz} = F_{zz}(R/\hat{\xi}, t/\hat{\xi})$$

scaling dimensions
the same as in the ground state

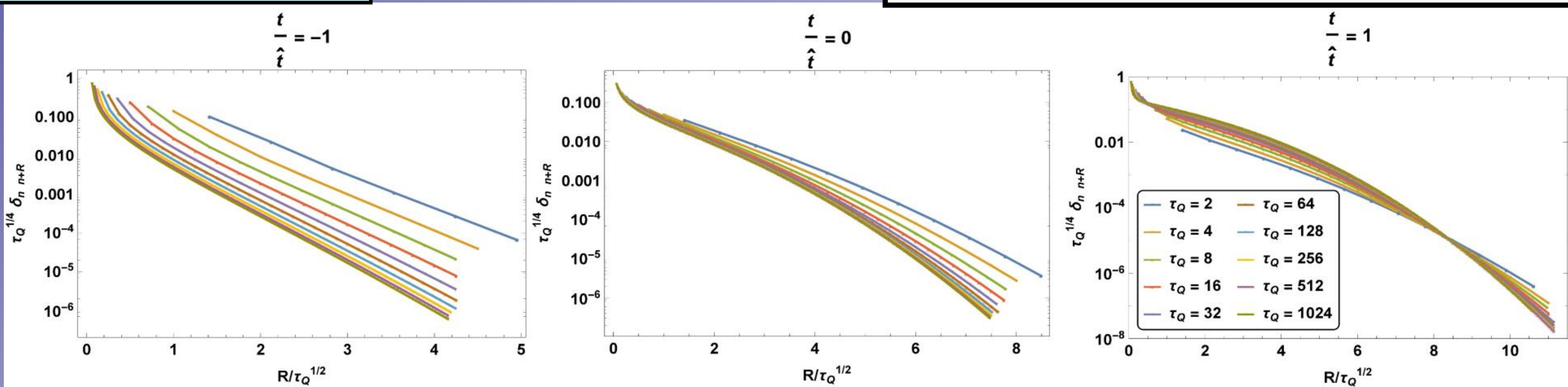
mutual information

$$\hat{\xi}^{1/2}\, I_R = F_I(R/\hat{\xi}, t/\hat{\xi})$$



quantum discord

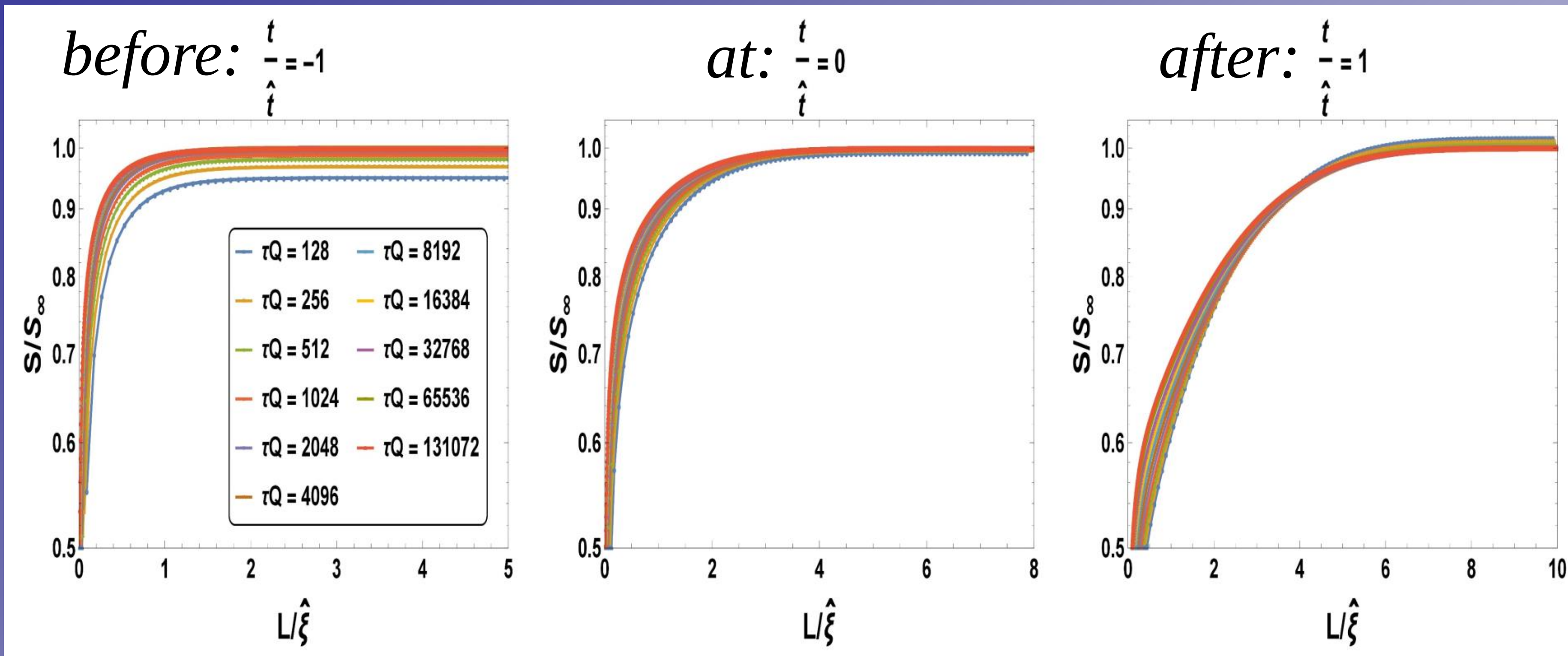
$$\hat{\xi}^{1/2}\delta_{\mathrm{R}} = F_\delta(R/\hat{\xi}, t/\hat{\xi})$$



$$\hat{\xi} = \sqrt{\tau_Q}$$

entanglement entropy: block of L spins

$$S_L = -\mathrm{Tr}\ \rho_L \log \rho_L$$



$$\frac{S_L(t)}{\frac{c}{3}\log k(t/\hat{\xi})\hat{\xi}} = F_S(L/\hat{\xi}, t/\hat{\xi})$$

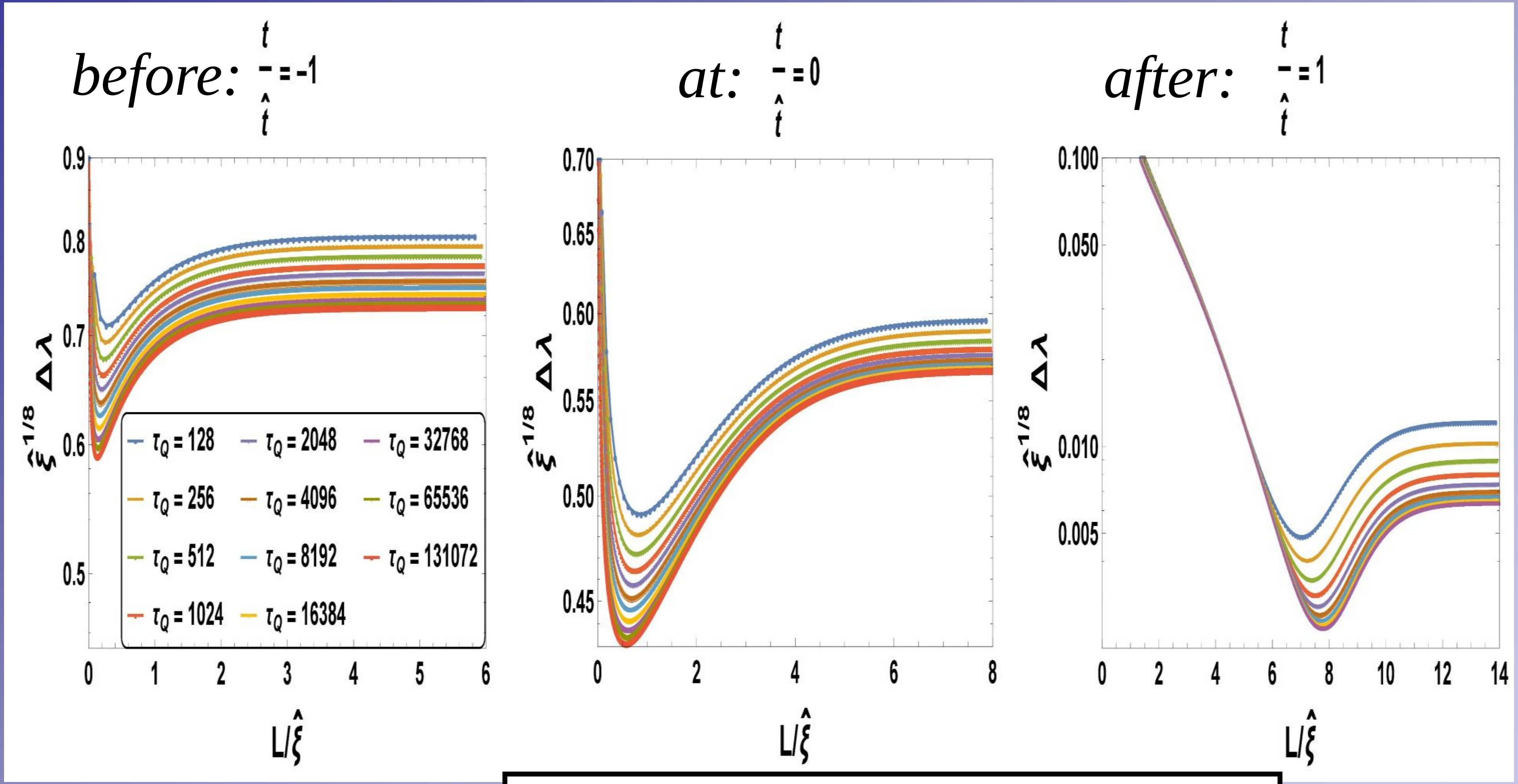
$$\hat{\xi} = \sqrt{\tau_Q}$$

central charge fits:

$$c = 0.52, 0.50, 0.48 \approx 1/2$$

entanglement gap: block of L spins

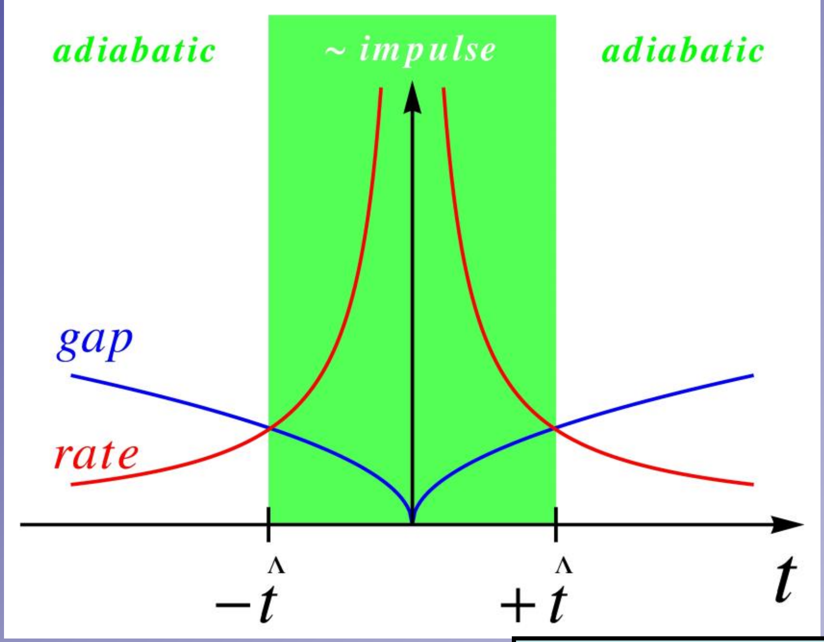
$$\Delta\lambda = \lambda_1 - \lambda_2$$



$$\hat{\xi}^{1/8}\,\Delta\lambda = F_{\Delta\lambda}(L/\hat{\xi}, t/\hat{\xi})$$

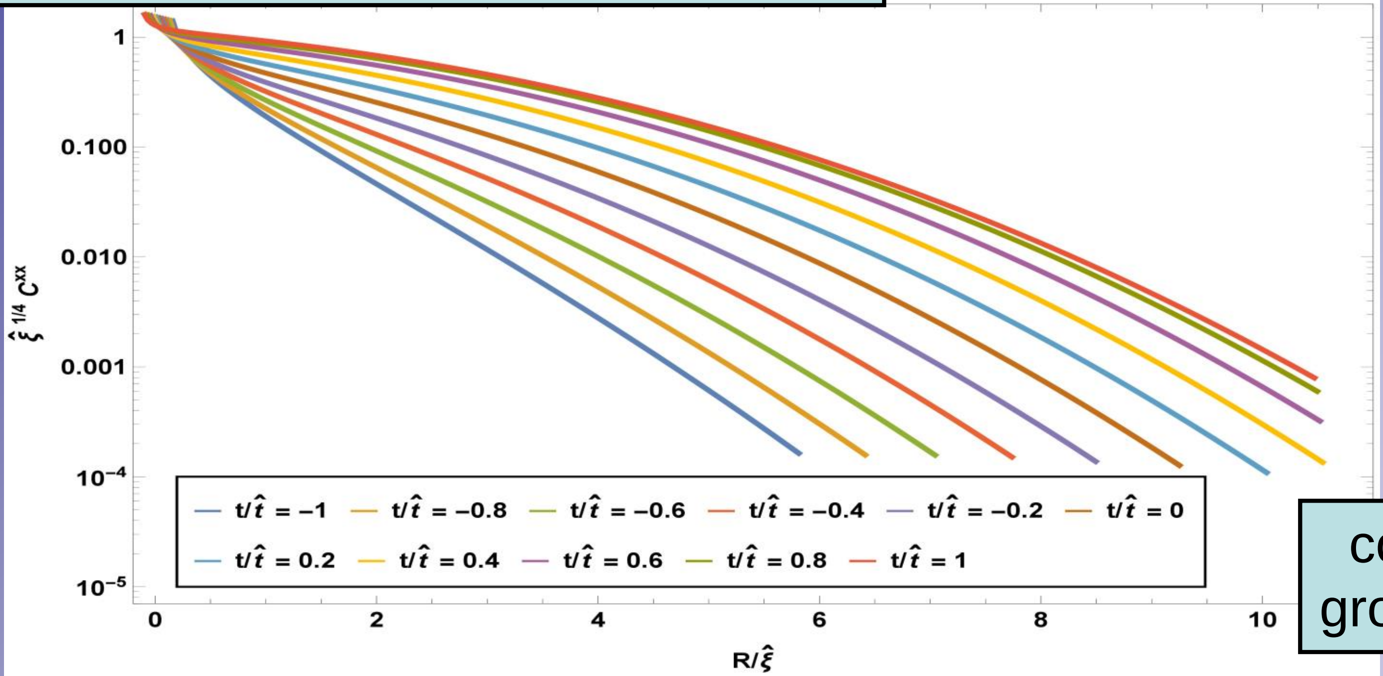
$$\hat{\xi} = \sqrt{\tau_Q}$$

beyond impulse approximation



cartoon version

ferro-correlator in the ``impulse'' stage



correlation range grows several times

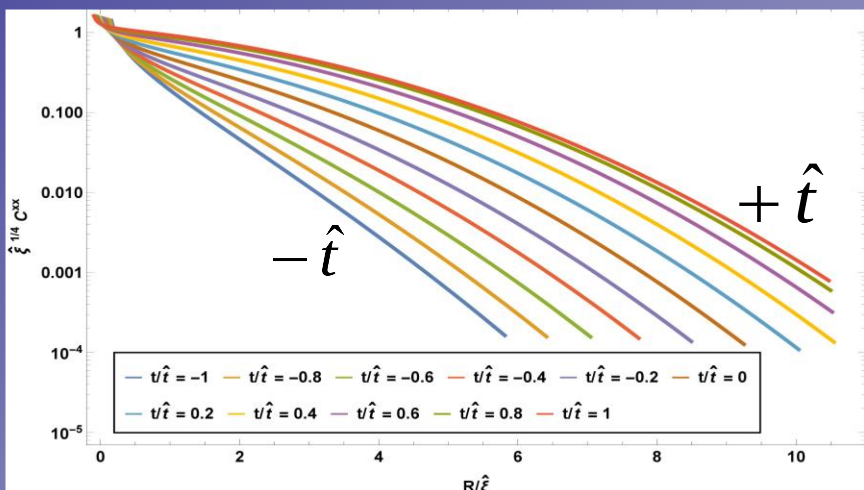
KZ sonic horizon

quasiparticle dispersion

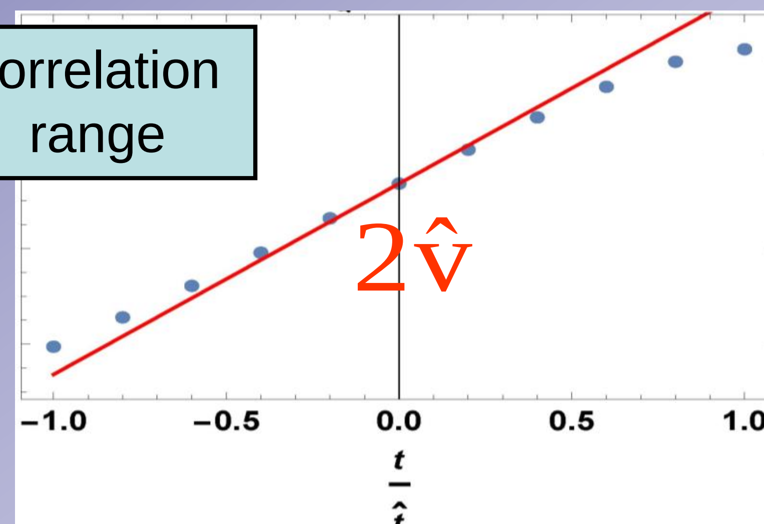
$$\omega \propto k^{z} \;\Rightarrow\; \mathrm{v} \propto k^{z-1}$$

maximal speed of sound

$$\hat{k} \propto \hat{\xi}^{-1} \Rightarrow \hat{\mathrm{v}} \propto \tau_Q^{-\nu(z-1)/(1+z\nu)} \propto \frac{\hat{\xi}}{\hat{t}}$$



correlation range

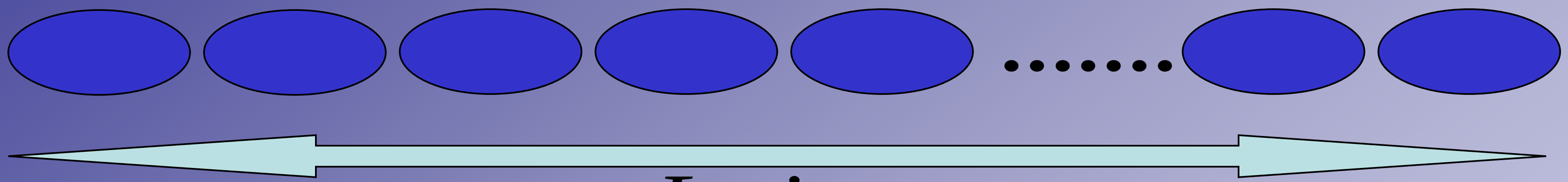


$$t \approx -\hat{t}: \quad \hat{\xi}$$

$$t \approx +\hat{t}: \quad \hat{\xi} + (2\hat{\mathrm{v}})(2\hat{t}) \approx 5\hat{\xi} \propto \hat{\xi}$$

1D Bose-Hubbard model: Mott -> superfluid transition

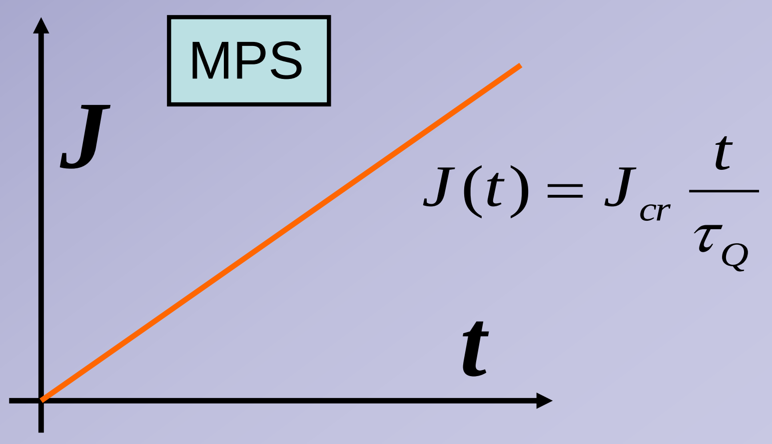
$$H = -J \sum_{s=1}^{L} \left(a_s^+ a_{s+1} + a_{s+1}^+ a_s \right) + \frac{1}{2} \sum_{s=1}^{L} a_s^+ a_s^+ a_s a_s$$



L sites

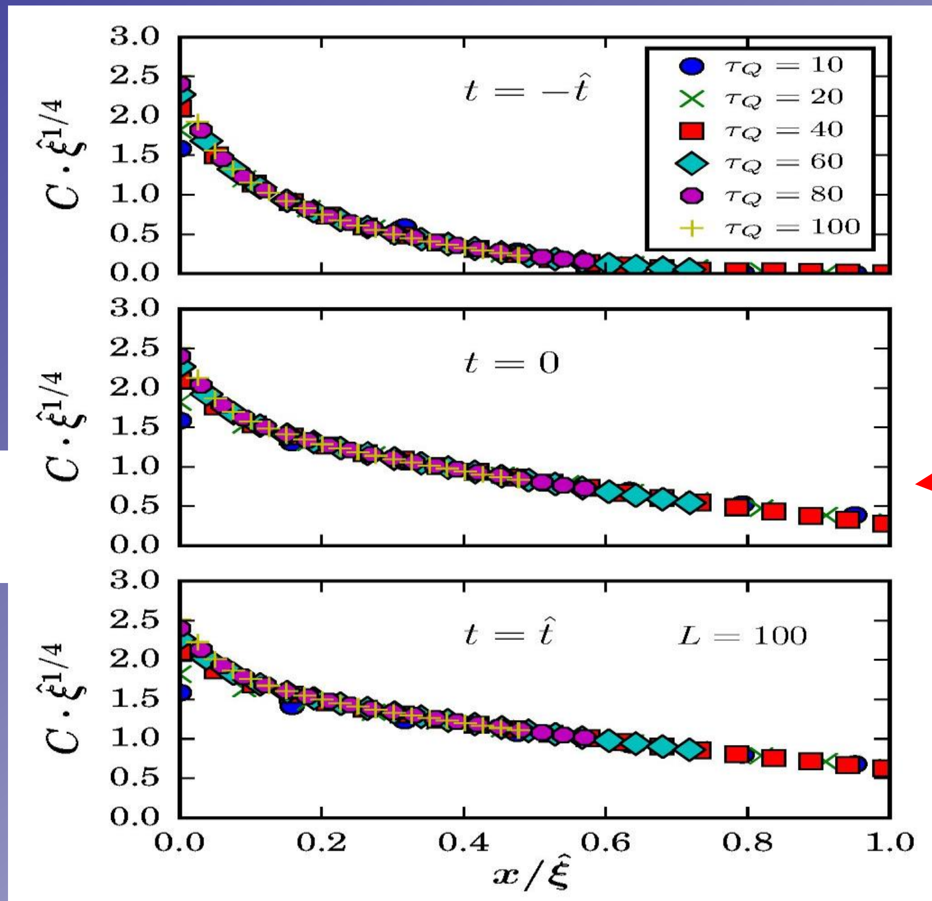
$J = 0$ at $t = 0$

$$|\psi(0)\rangle = |1,1,1,1,1,\ldots.,1,1\rangle$$



1D Bose-Hubbard model: correlation function

$$C_R = \langle a^{+}_{s+R} a_s \rangle$$



$\hat{\xi}^{1/4} C_R$

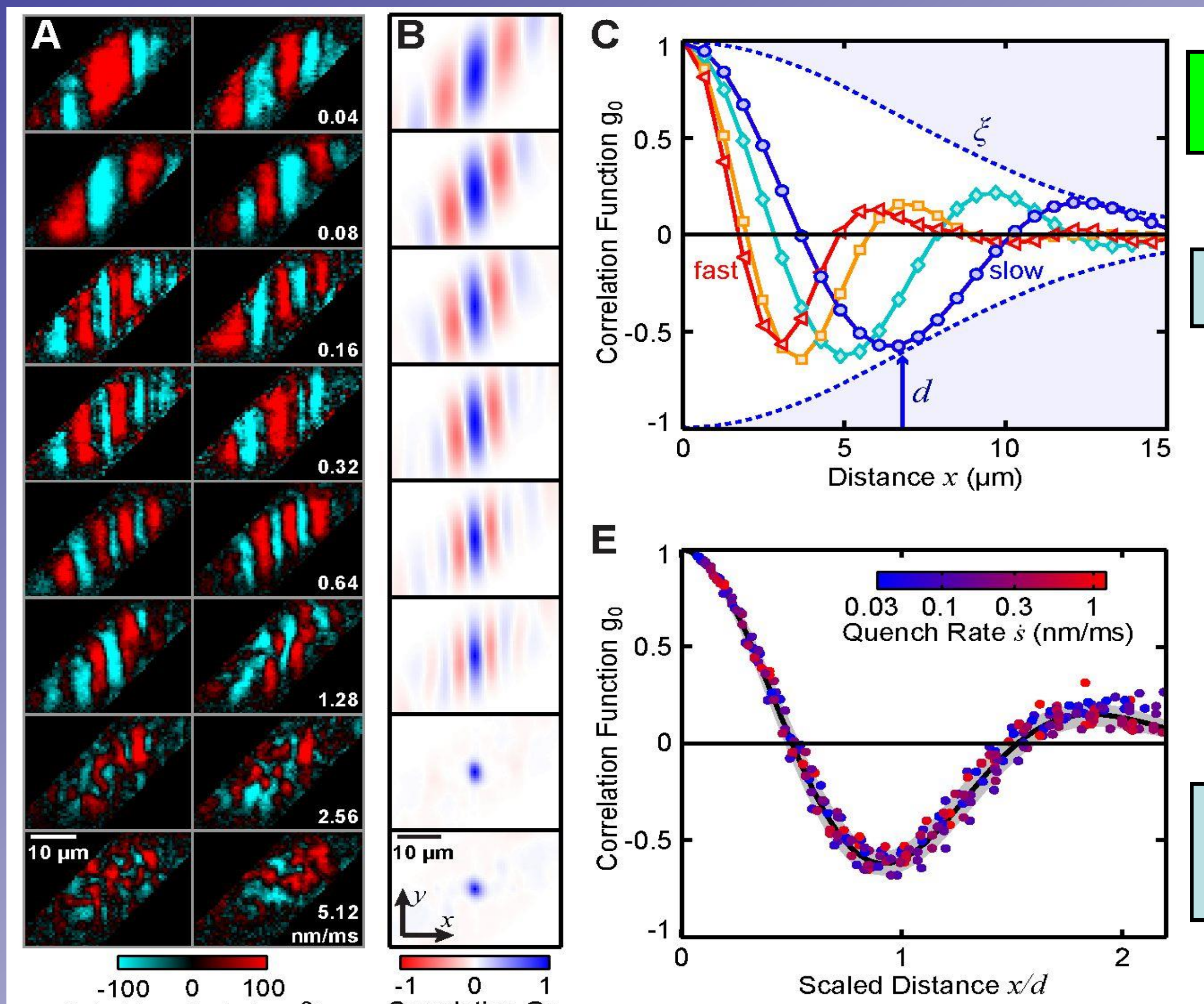
Kosterlitz-Thouless

$\hat{\xi} \propto \tau_Q^{0.8}$

previous simulations & experiment

$\tau_Q << 10$

Experiment:
``Universal space-time scaling symmetry in
the dynamics of bosons across a quantum phase transition”



Chin’s group, Science 2016

correlator

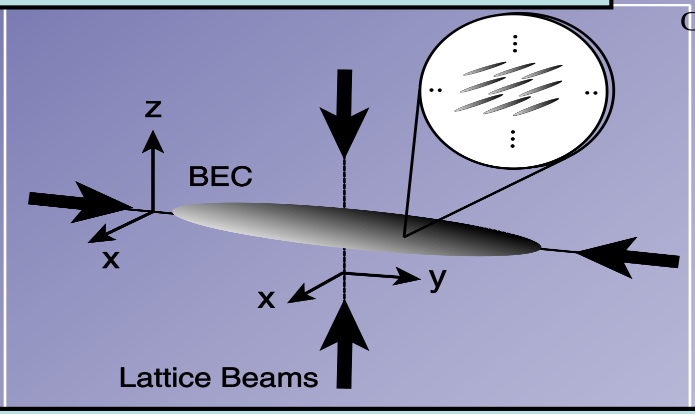
scaled correlator

HOMEWORK:

1) Cold atoms:

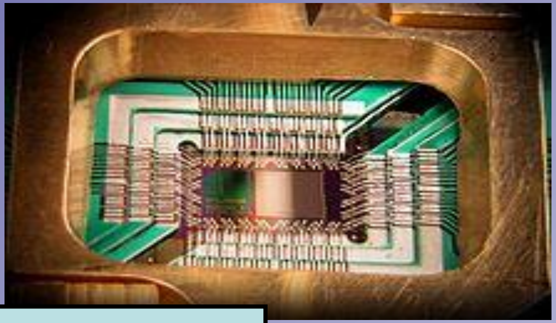
Mott->superfluid with n=1 atom per site:
not integrable, not mean-field

classical supercomputer



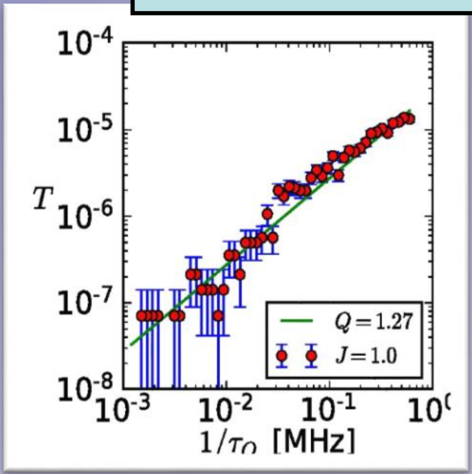
perfect job for a quantum simulator

2) D-Wave:



~ 2000 qubits

scaling law due to
decoherence, not KZ



Martinis @ Google:
only 7x7 qubits
but
better isolated (?)
Ready way to test it